Week 2 Lecture Notes

Solar-Terrestrial Physics –

The Sun’s Atmosphere, Solar Wind, and the Sun-Earth Connection

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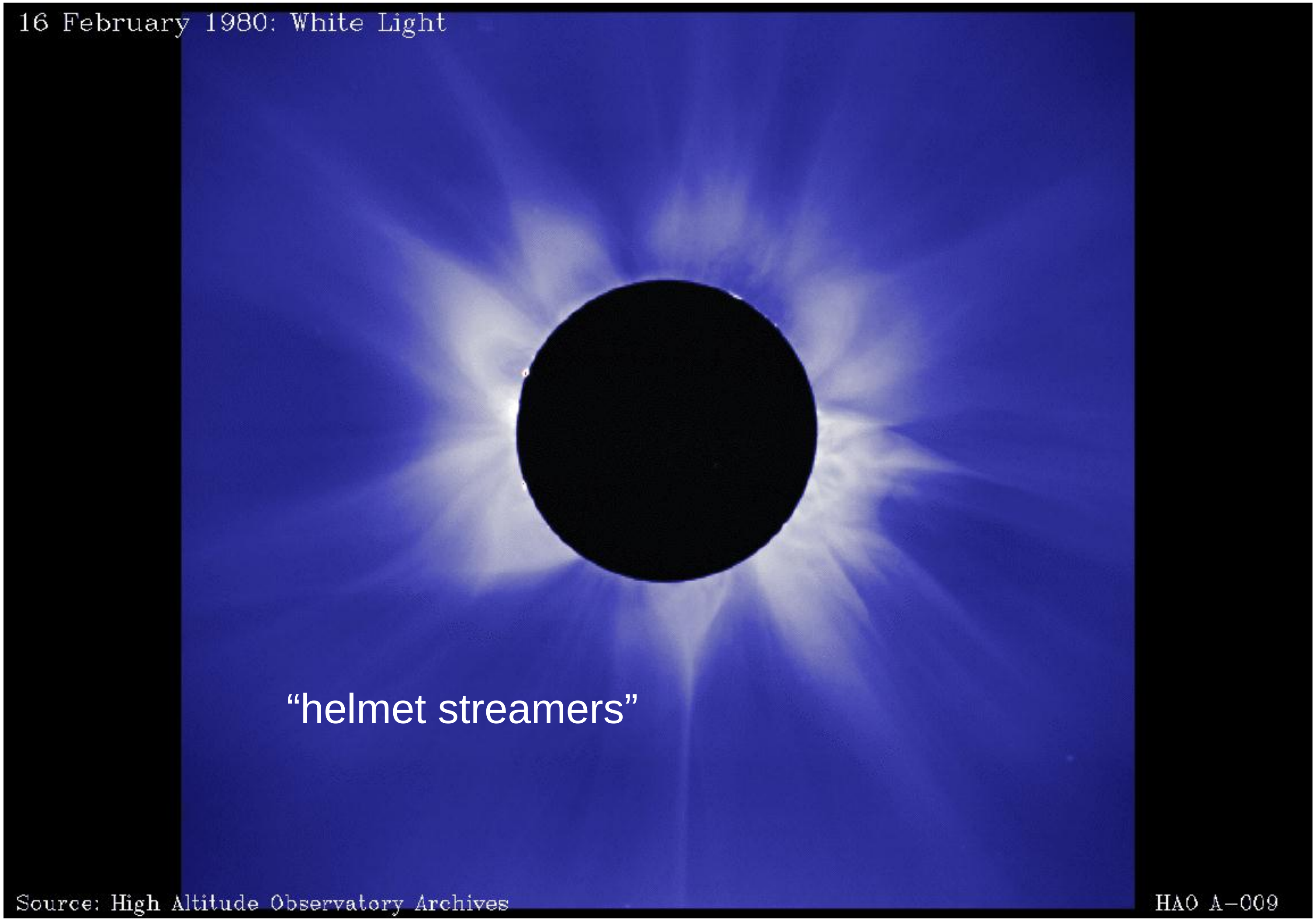
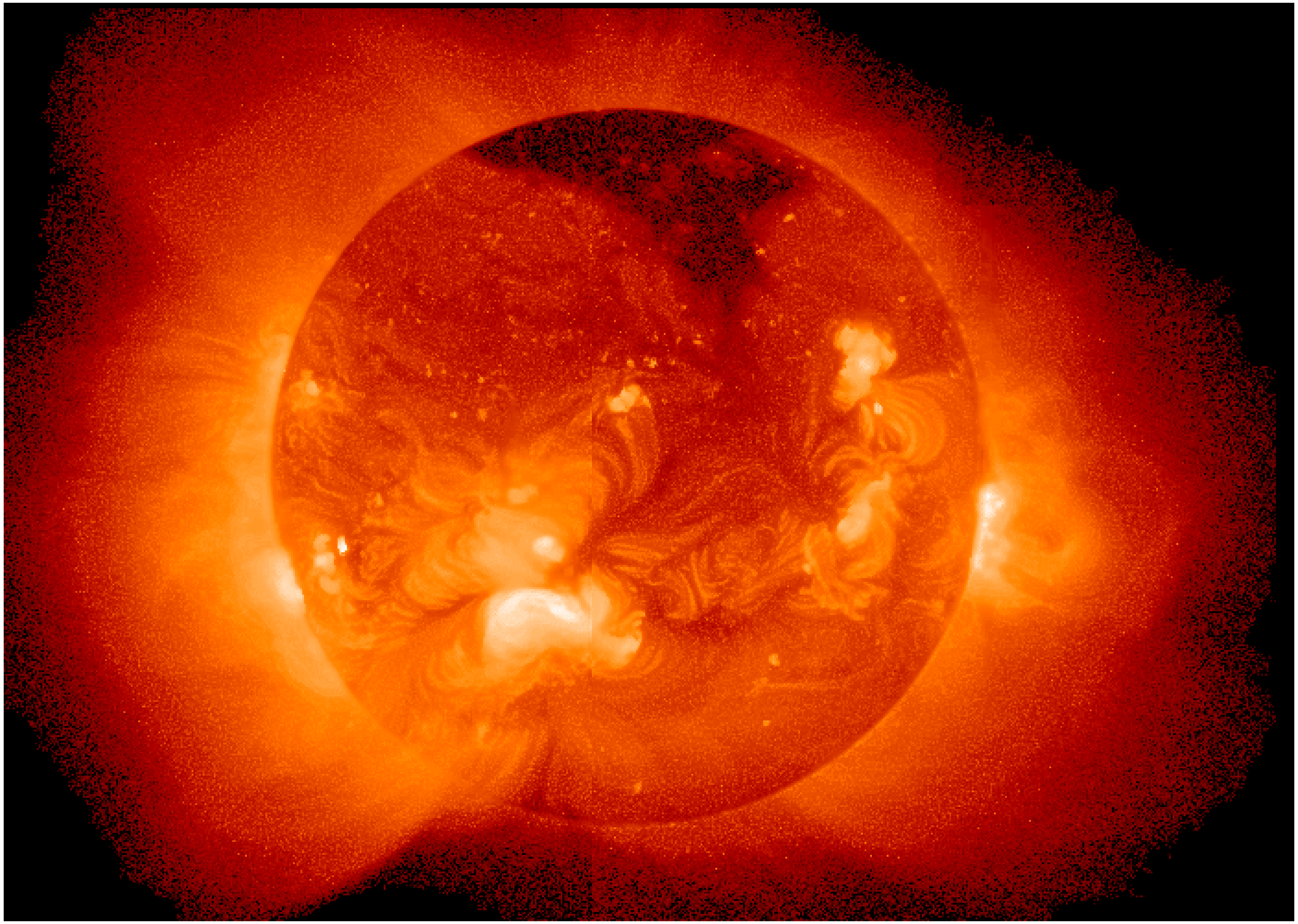
The Solar Corona is the Sun’s Extended Atmosphere

Scattered light makes it visible during a total eclipse of the Sun



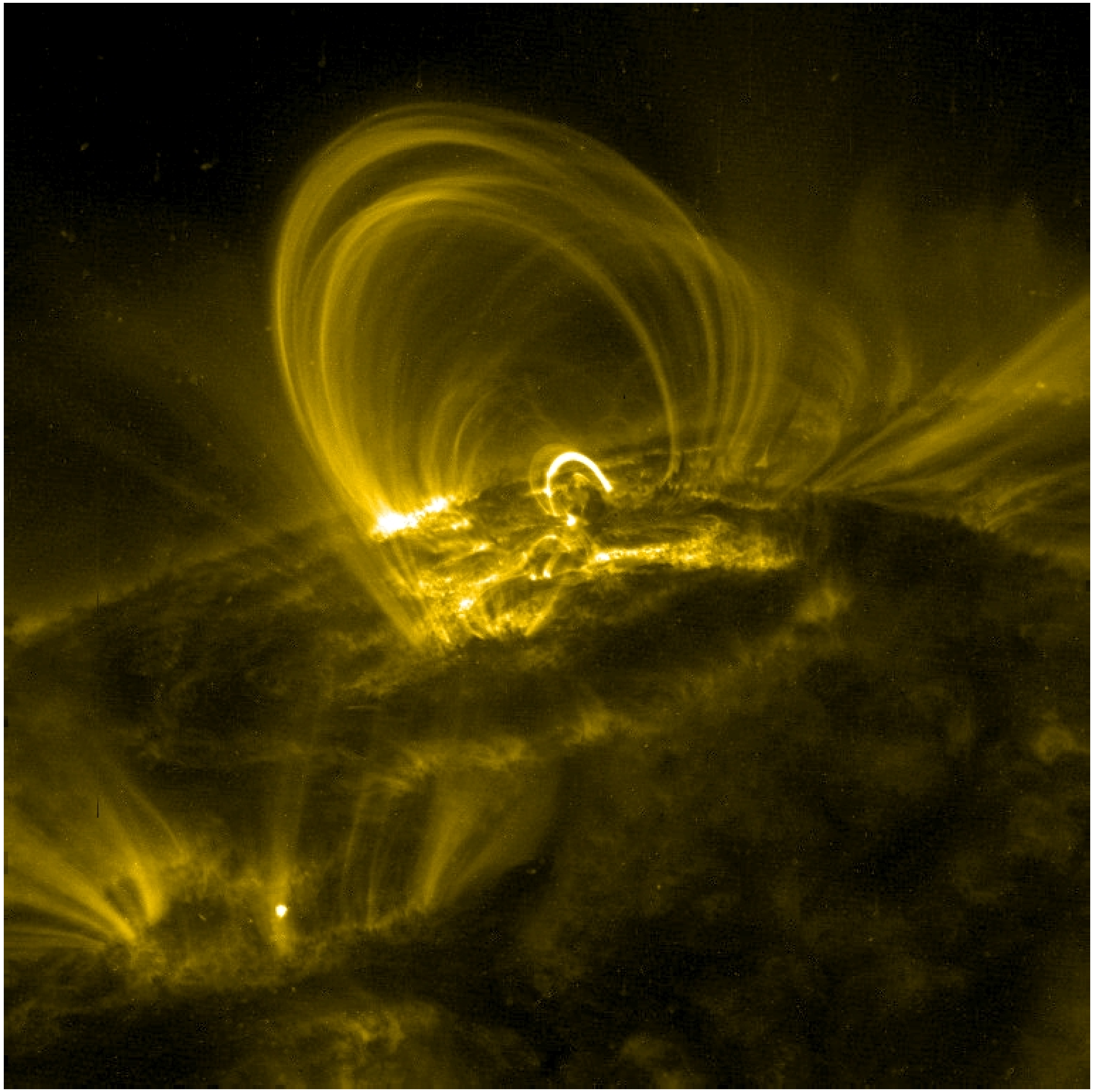
X-Rays Reveal 3D Magnetic Loops and Arches

The corona is full of magnetic structures at all scales



http://www.lmsal.com/SXT/homepage.html

Close-Up of Some Magnetic Loops

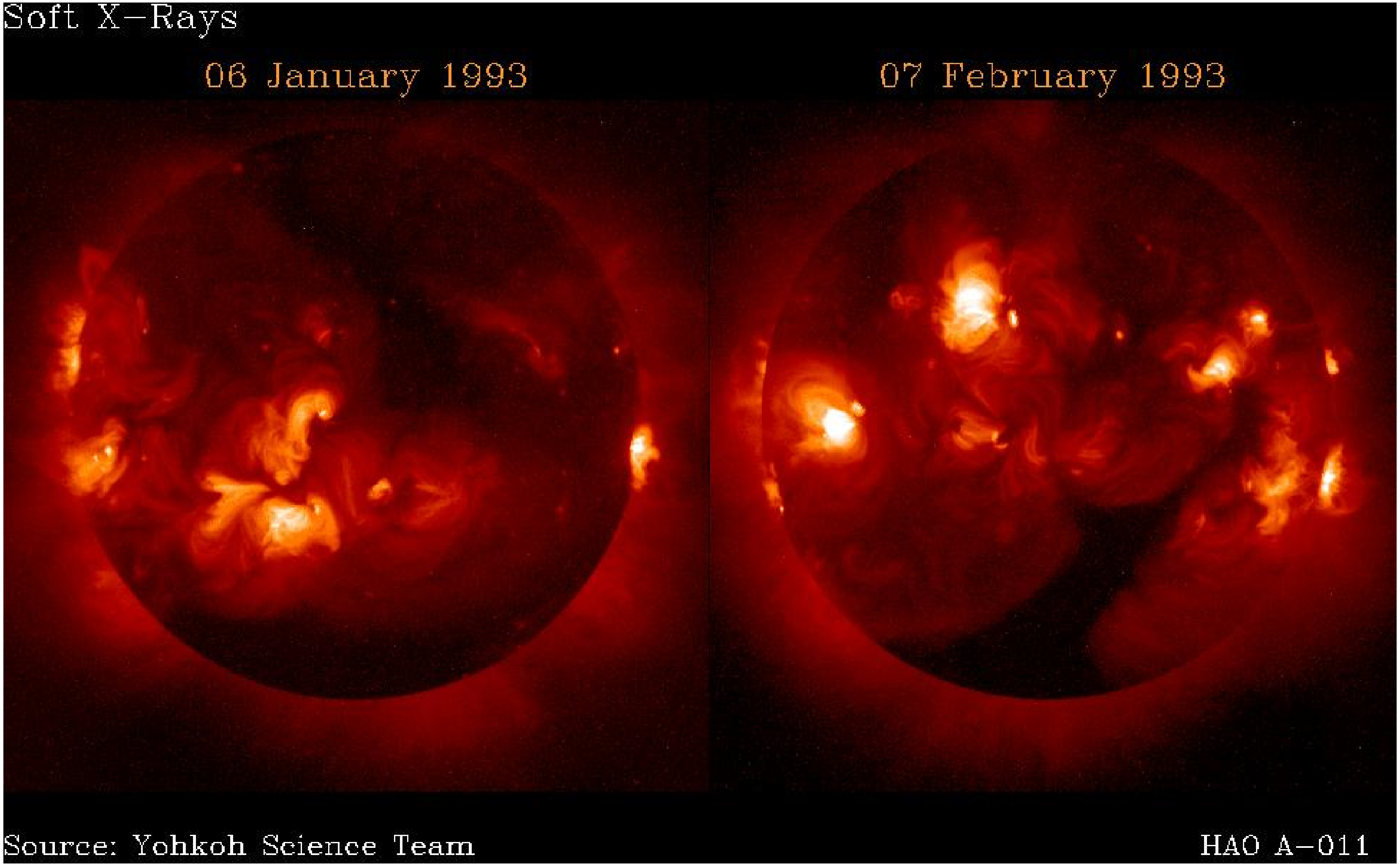


Data from the TRACE satellite at 171 Å (EUV)

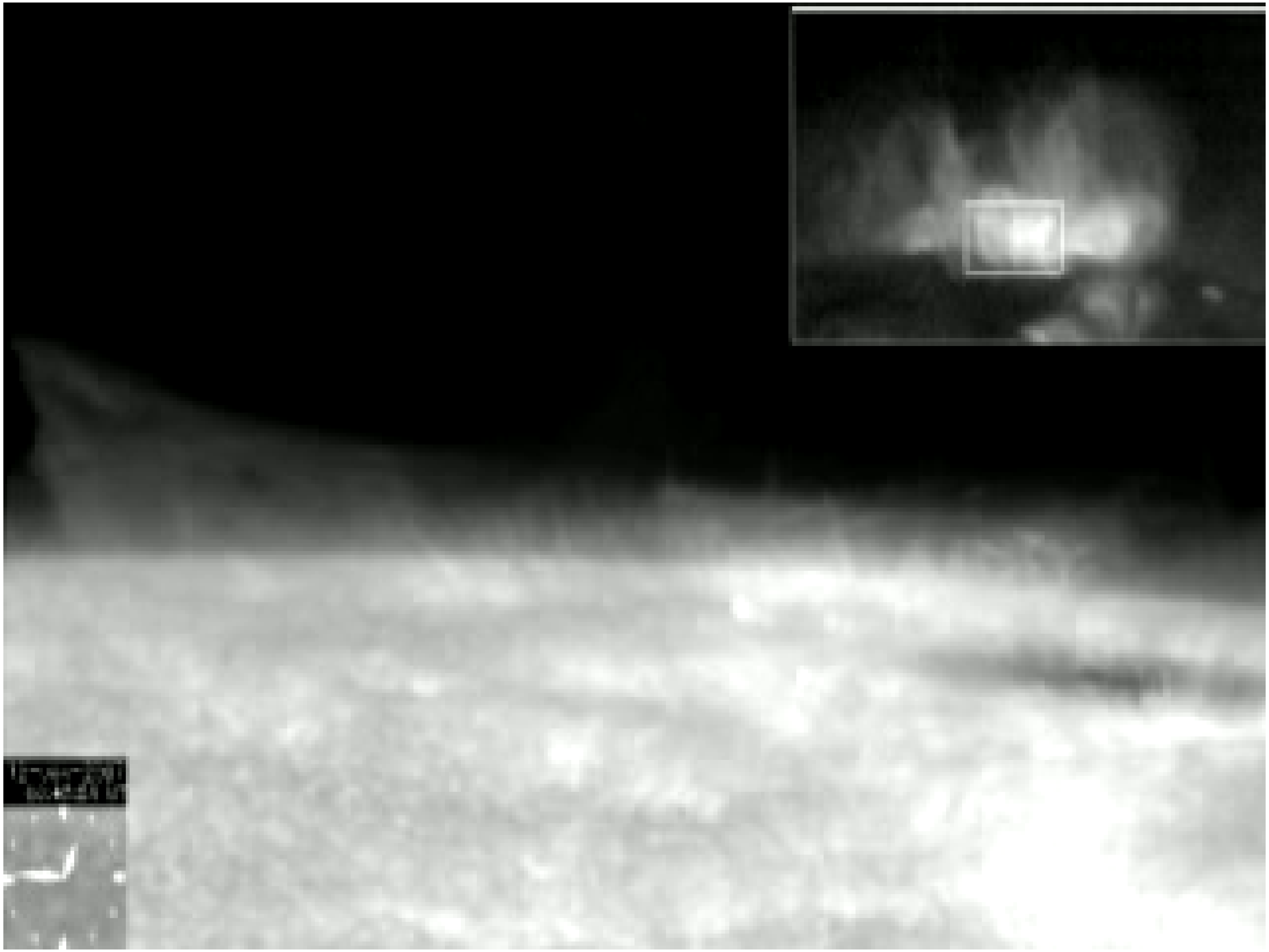
**QuickTime movie of Yohkoh SXT images shows the 3D structure of magnetic loops*

Coronal Holes

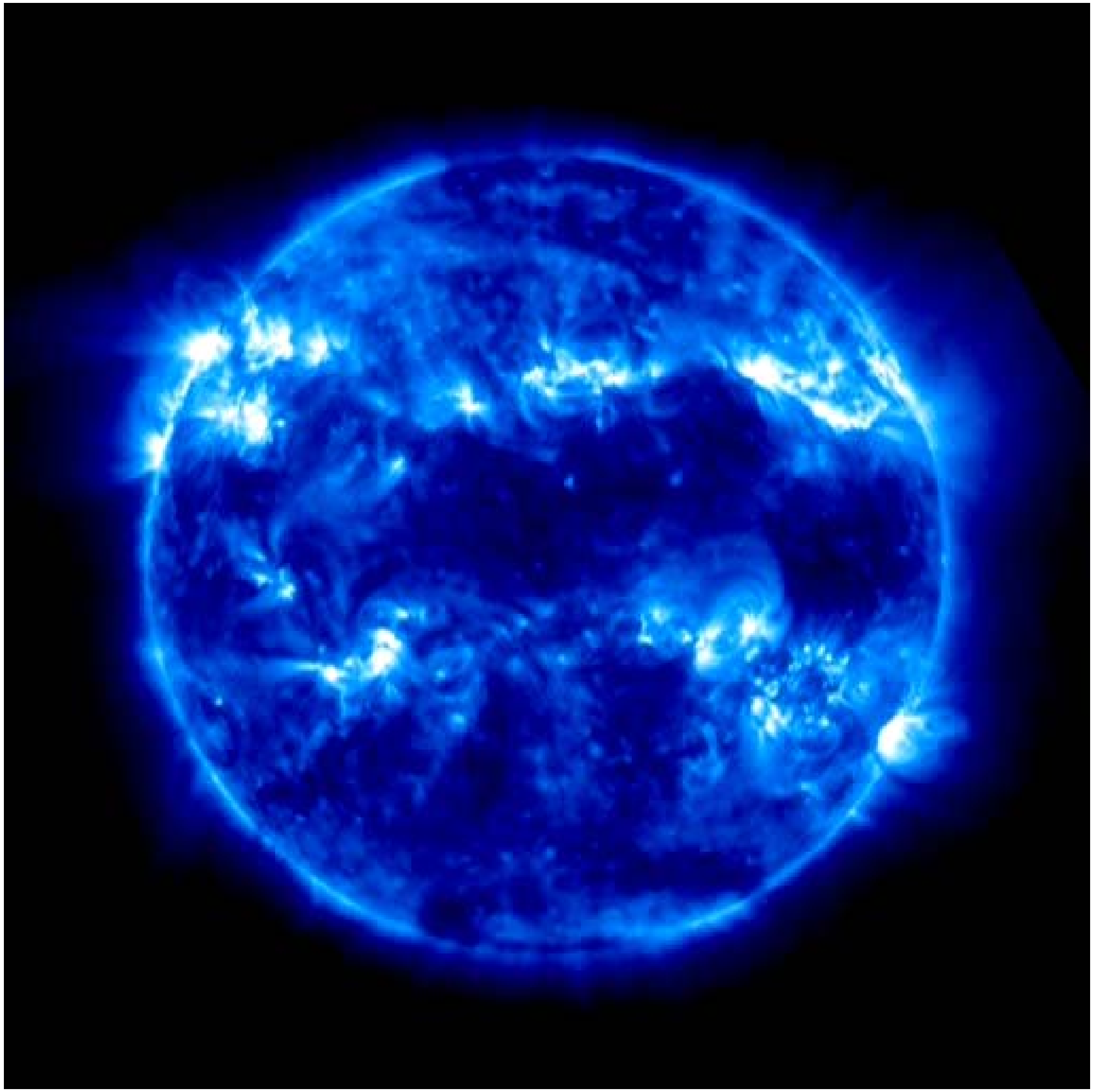
Usually found at the poles, they can extend to lower latitudes



The Corona is a Very Dynamic Place!

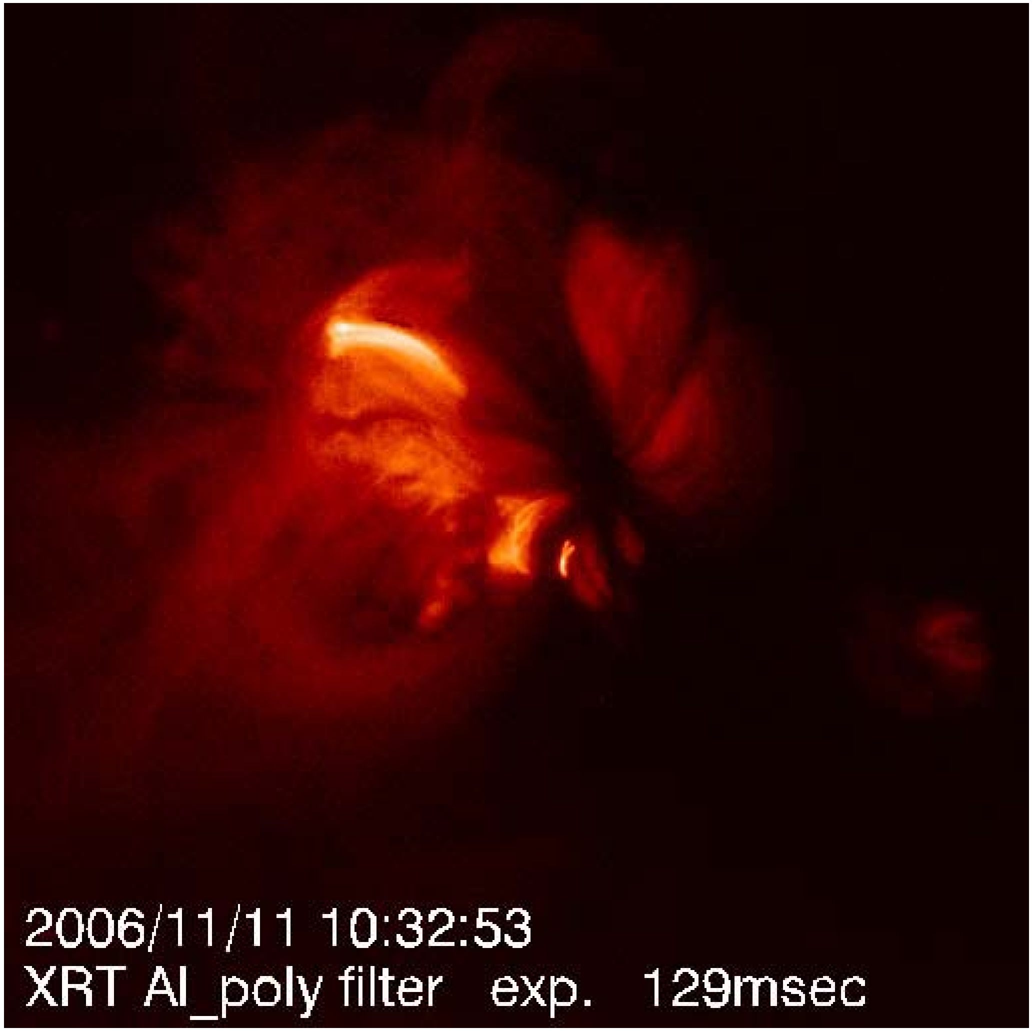


The Restless Corona (from SOHO)



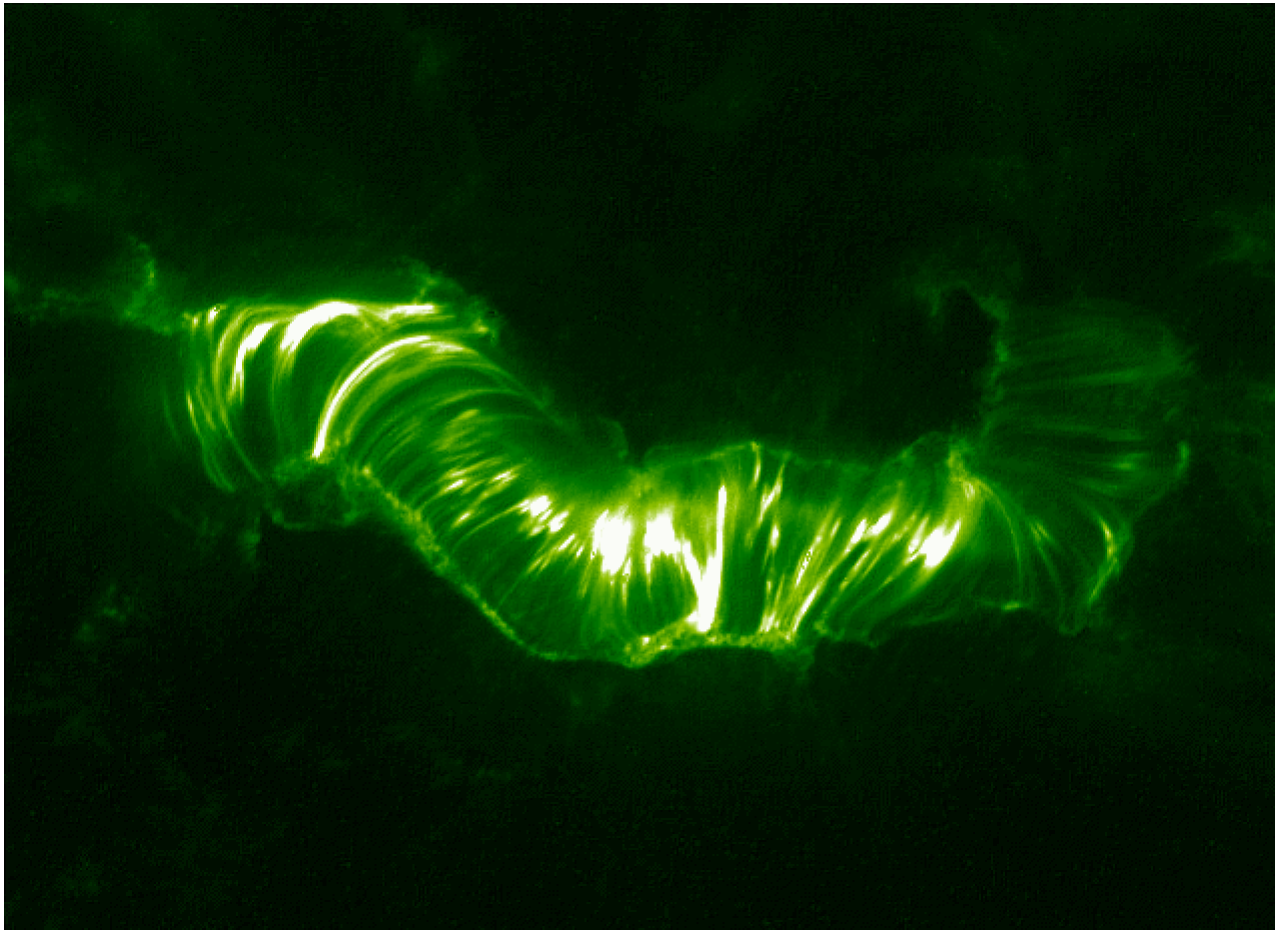
Solar Flares

Plasma catastrophes trigger bursts of radiation



Flares Often Occur Along Coronal Arcades

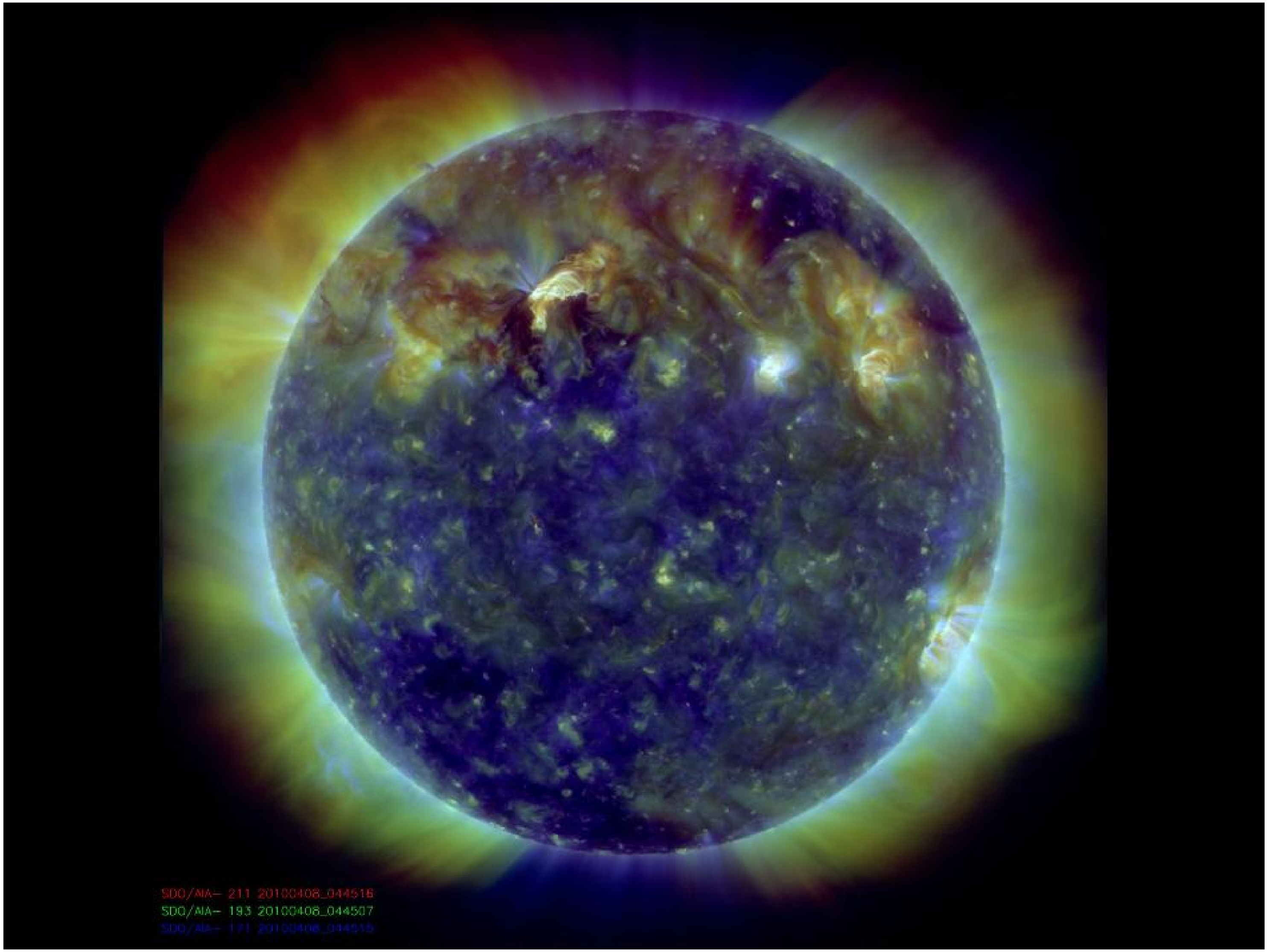
An arcade marks a seam between regions of opposite polarity.
Shear motion along the seam can cause it to flare all at once.



(TRACE image)

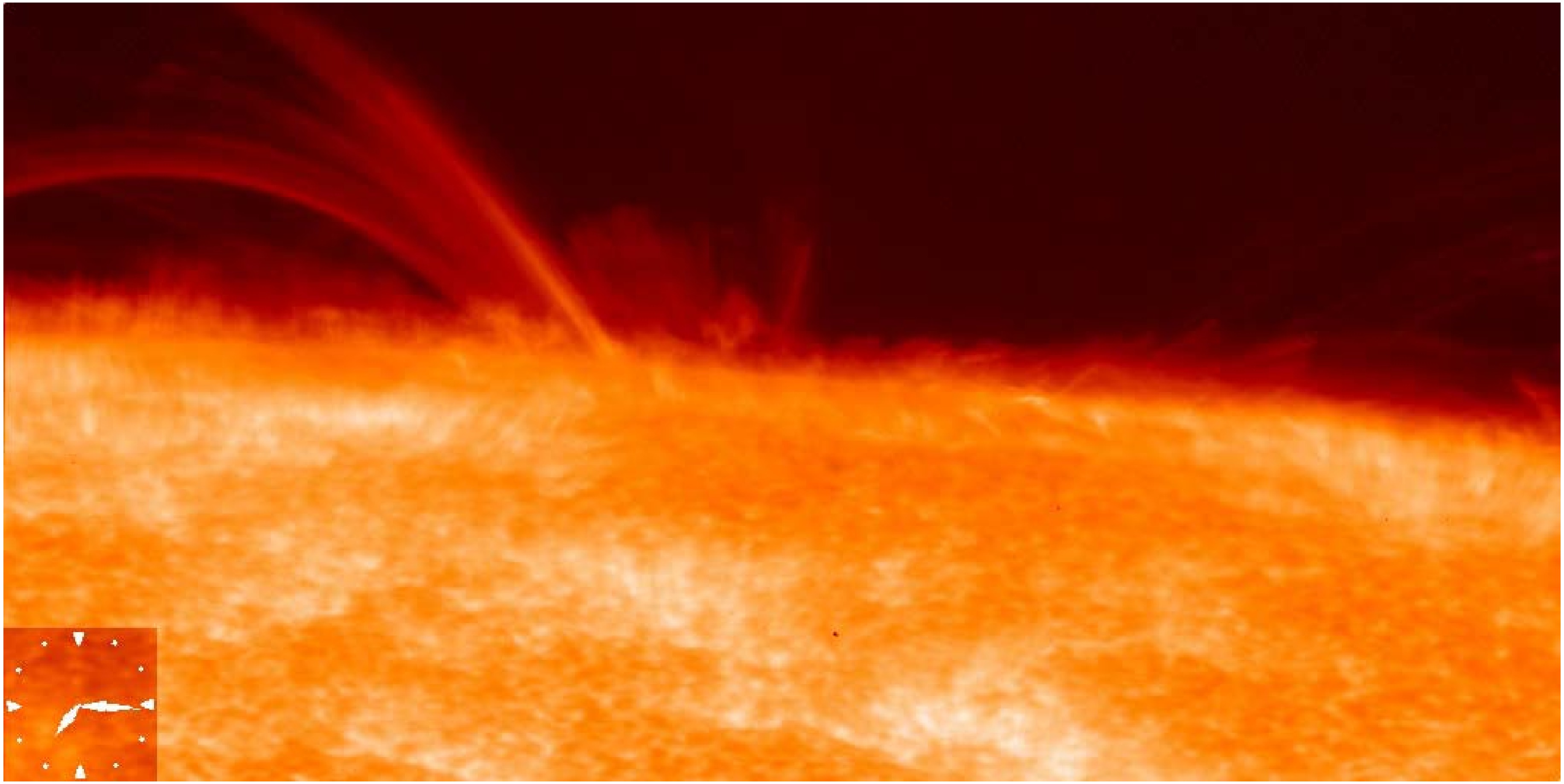
Flare Movie from SDO

First one flash, then more, then a shock that rearranges the global field!



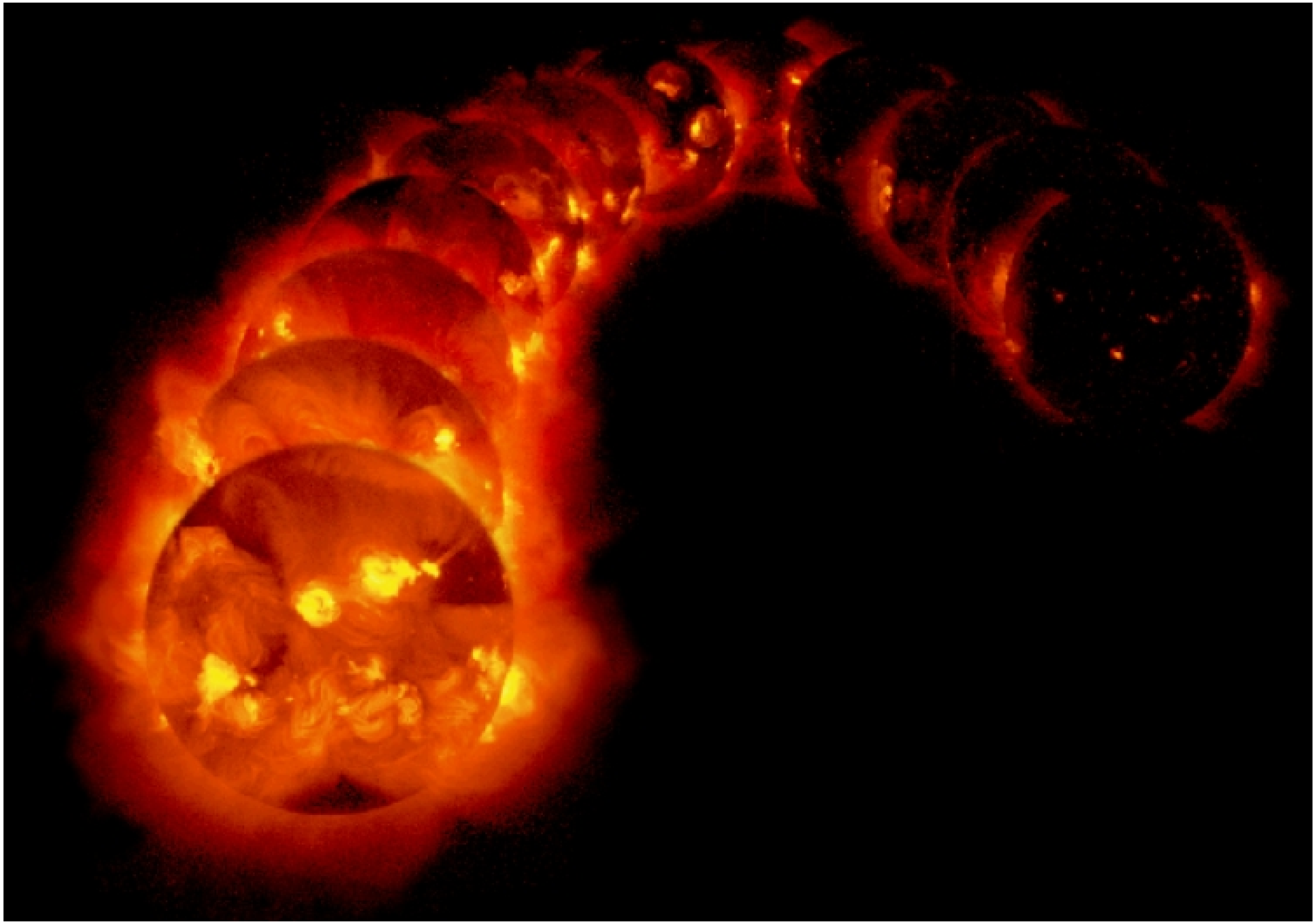
**See the sequence in the QuickTime movies*

Flares Are Also Associated with Flux Emergence



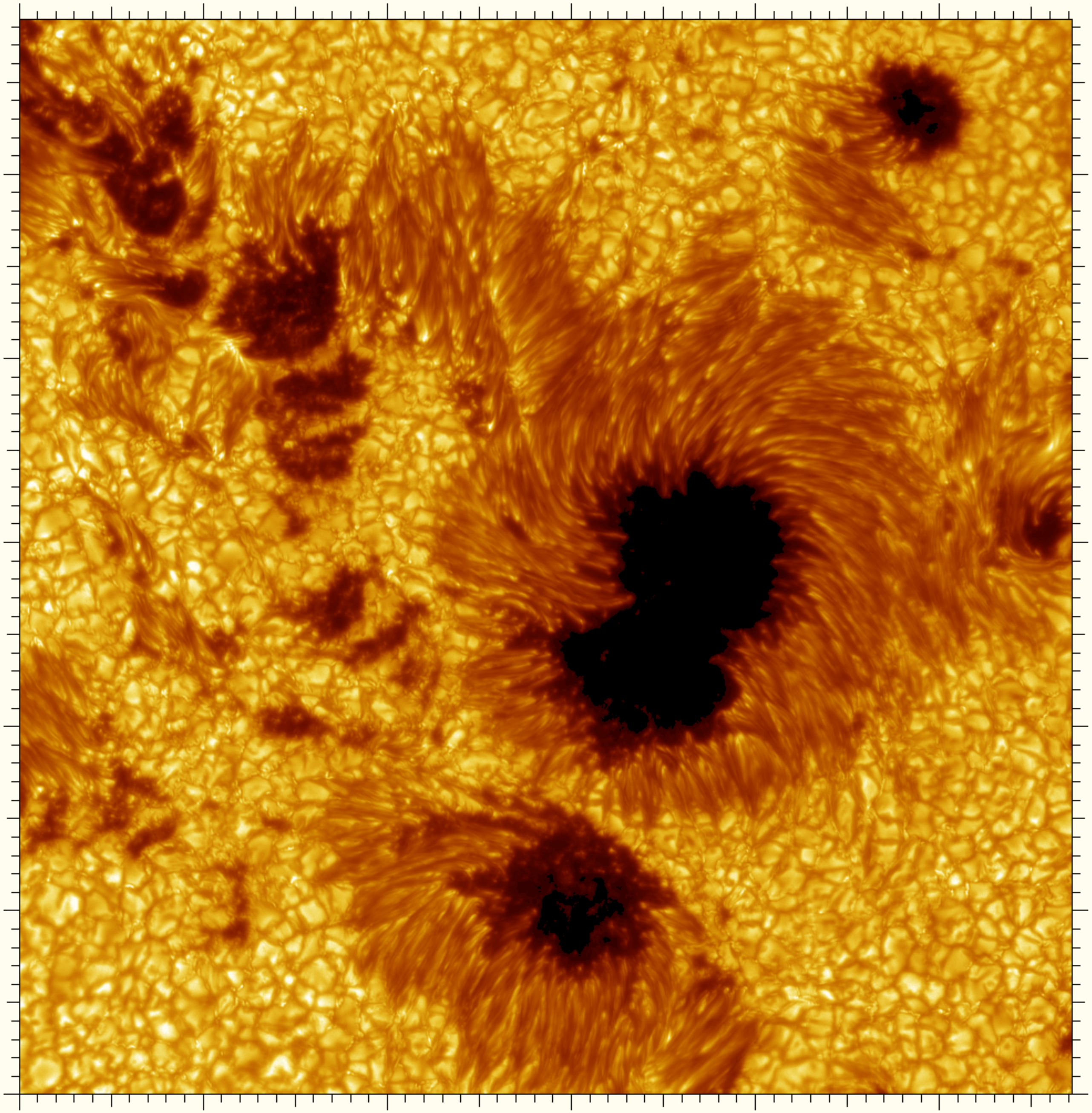
Hinode initial results page: http://solar-b.nao.ac.jp/news_e/20061127_press_e

The Corona in X-Rays from Solar Max to Min



http://en.wikipedia.org/wiki/File:Yohkoh_solar_cycle.jpg

Sunspots and Active Regions

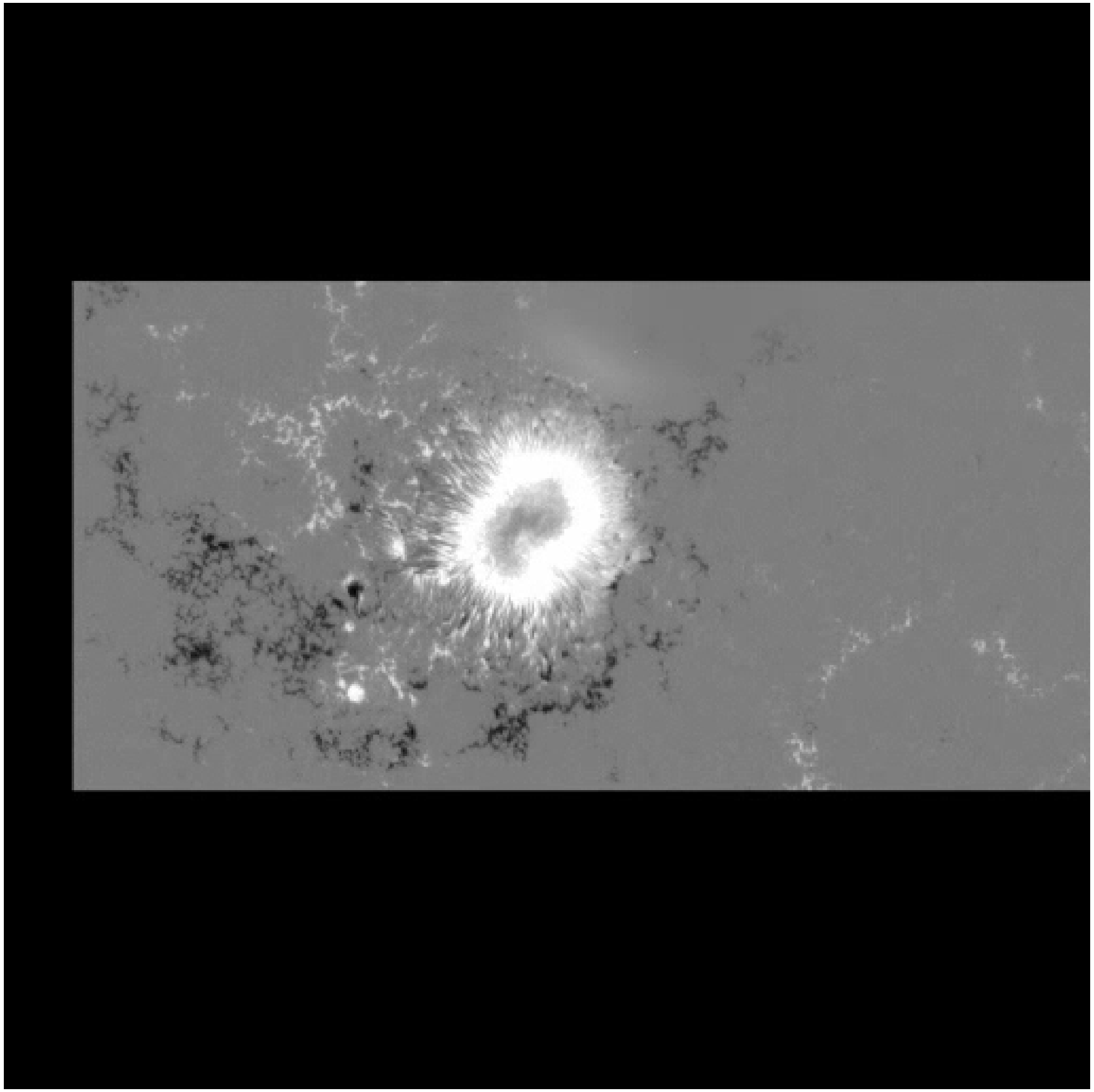


This was the most highly resolved solar image ever taken by the 1-meter Swedish Solar Telescope (SST) on La Palma.

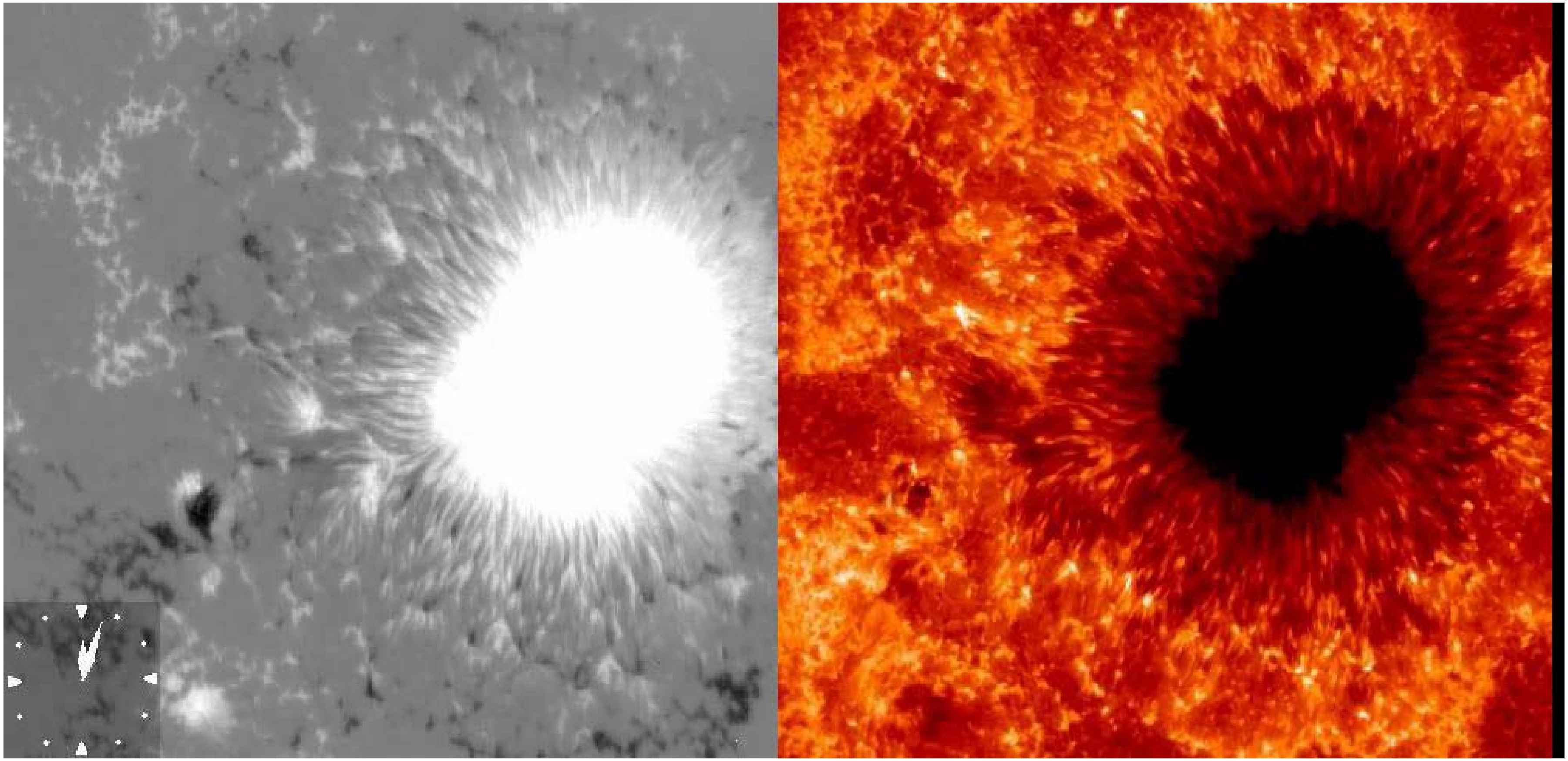
- Dark patches: umbrae
- Less-dark streaks: penumbrae

Credit: Royal Swedish Academy of Sciences, 2002

X-Ray Emission Above a Sunspot



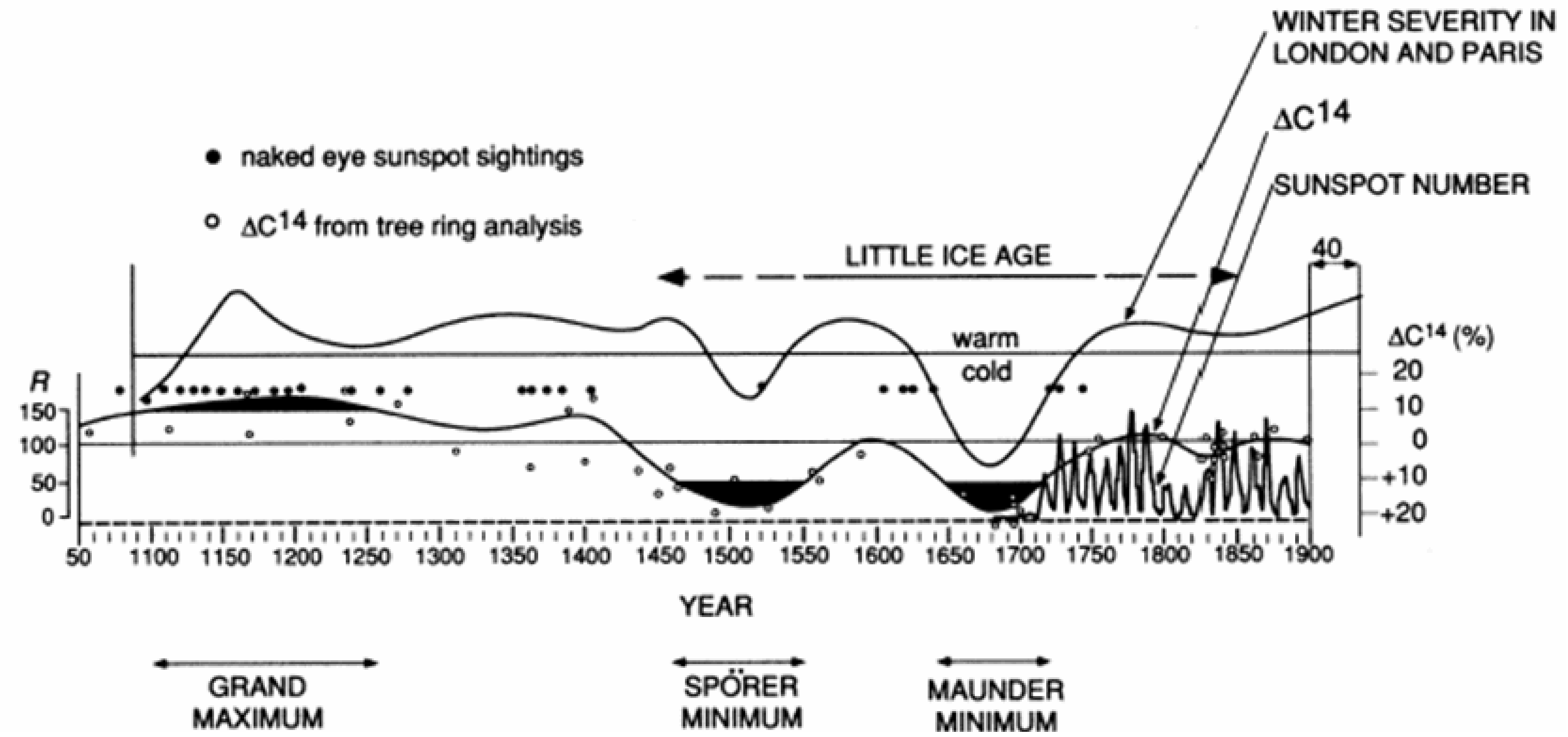
Evershed Flow in a Sunspot Penumbra



The *chromosphere* lies just above the photosphere. Here, magnetic features are highlighted by spectral lines like Hα, Ca II K, and Ca II H (image at right). When viewed in Hα, bright areas near sunspots are called “plage” (French for “beach”).

Sunspot Number May Influence Terrestrial Climate

- More sunspots means *more* light—bright *faculae* (“little torches”) outweigh dark sunspots. Rough explanation: toward the limb, strong magnetic fields create a sort of window into the deep, hot sides of convection cells.
- Just one more reason why understanding solar magnetism is important

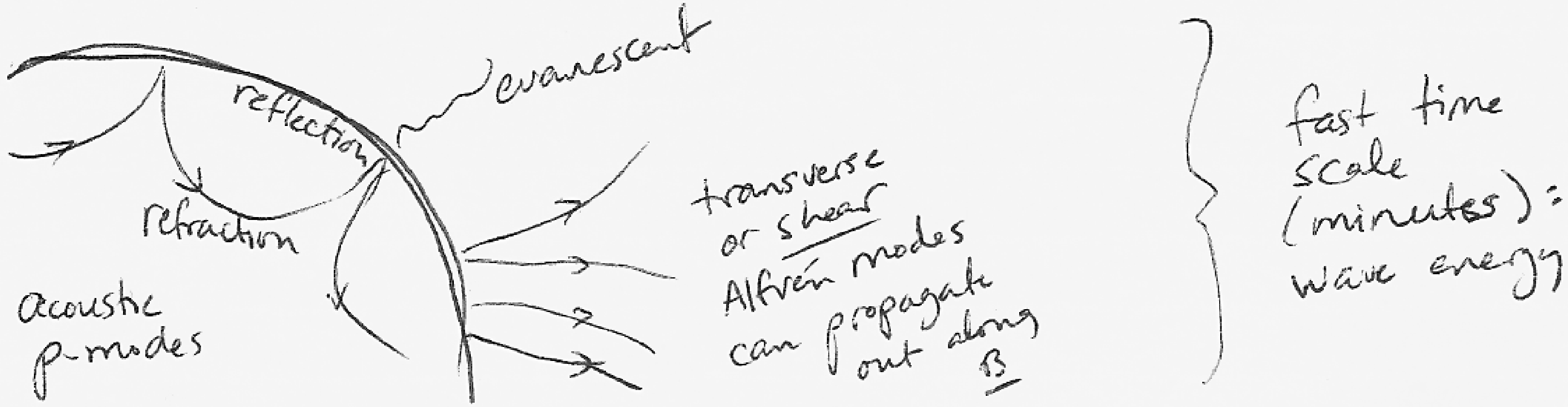


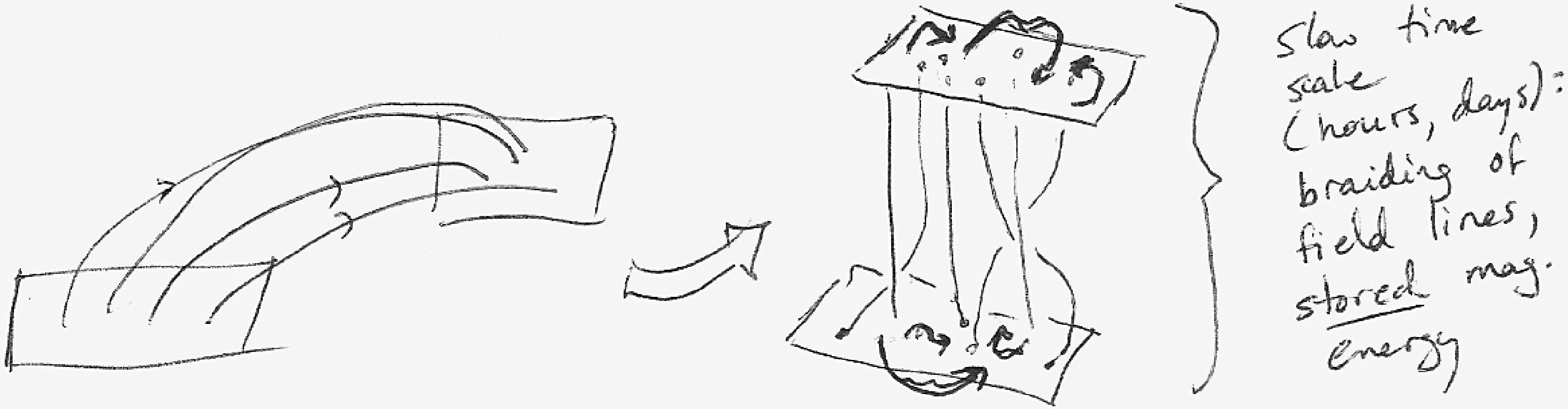
The Solar Corona

Why is the corona hot?

- Observation: coronal radiation implies very high temperatures
  - Unusual spectral lines can be traced to highly ionized atoms, e.g., Fe XIV
  - The corona is bright in X-rays with an equivalent blackbody temperature ~10^6 K
- Heat cannot just flow to a region of higher temperature
  - Violates the 2nd law of thermodynamics!
- Something must be doing mechanical work on the plasma
  - Magnetic energy is dominant in the corona
  - Work can be done *against* Lorentz forces to build up magnetic energy further
  - Ohmic heating of the plasma occurs where current is flowing
  - Points to a heating mechanism mediated by magnetic fields
- Two possible scenarios:
  - Waves from the photosphere (and below) travel up along the magnetic field, depositing energy as they go
  - Flares, microflares, nanoflares… solar flares of all scales are always happening, leading to magnetic reconnection and heating

Competing Models of Coronal Heating

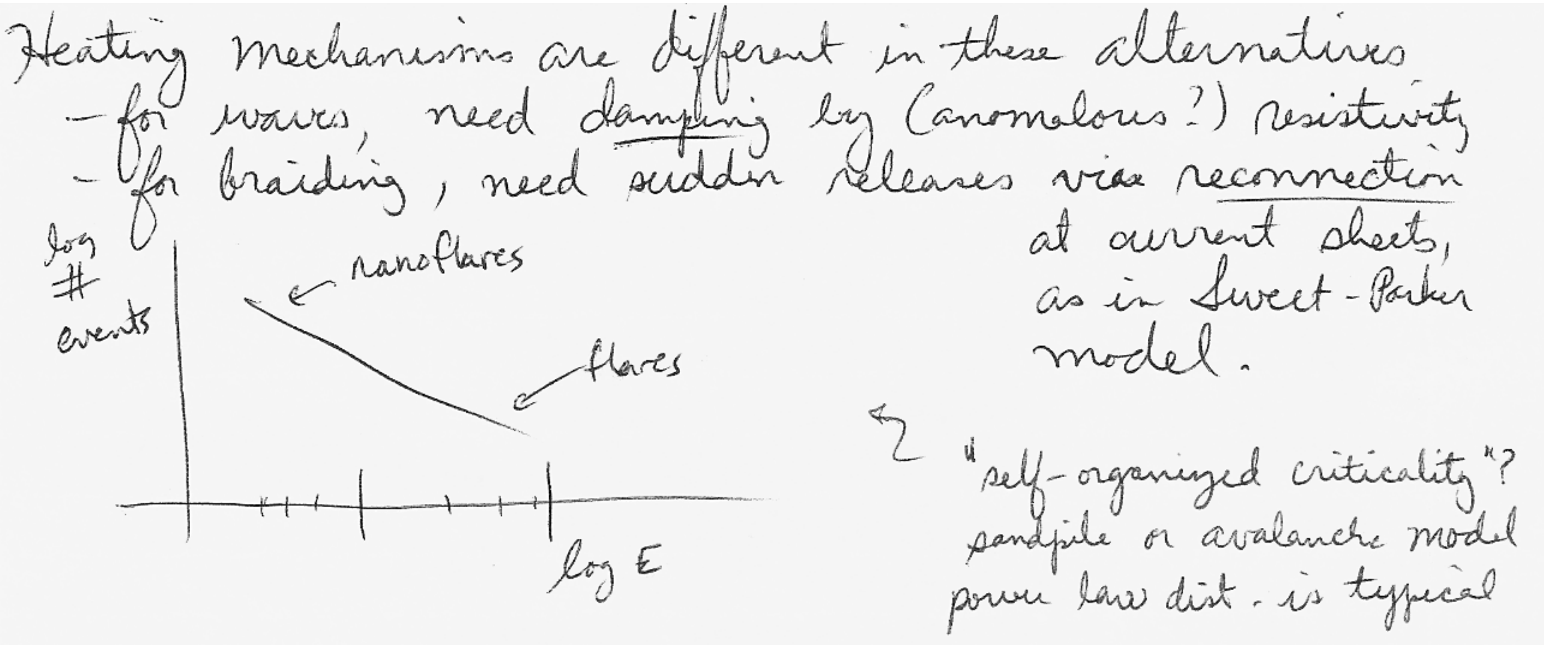




Problems with the Heating Models

Due to the low resistivity of the corona

- The corona makes a very good cavity for trapping waves, but not for dissipating them.
  - Magnetosonic waves don't propagate up through the chromosphere.
  - Shear Alfven waves propagate but are scarcely damped.
- Reconnection rates are slow. Nanoflares are not (yet) observed.



Heating mechanisms are different in these alternatives

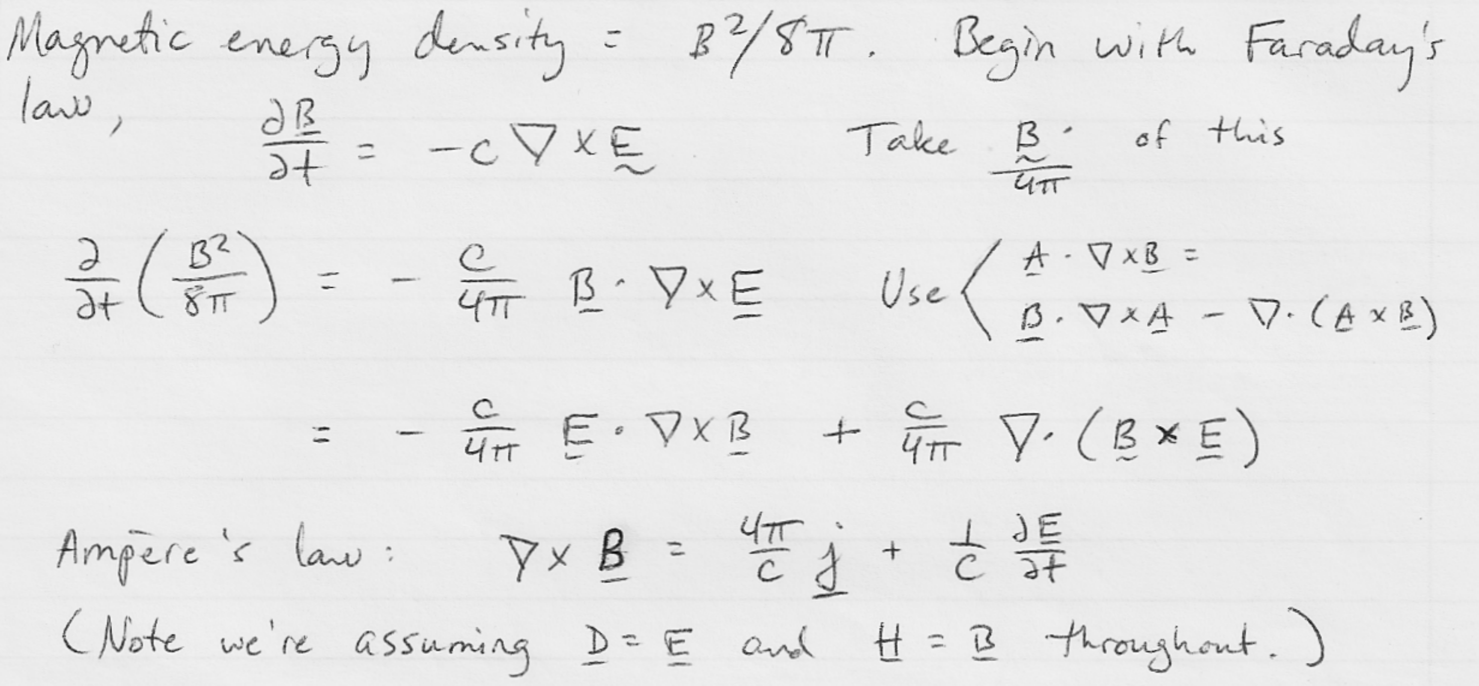
– for waves, need damping by (anomalous?) resistivity

– for braiding, need sudden releases via reconnection at current sheets, as in Sweet-Parker model.

"self-organized criticality"? sandpile or avalanche model power law dist. is typical

MHD Magnetic Energy Equation – 1

The starting point is the full electromagnetic energy equation, with no approximations, which can be derived from Maxwell's equations.



Magnetic energy density $= B^2/8\pi$. Begin with Faraday's law,

$$\frac{\partial \underline{B}}{\partial t} = -c\,\nabla \times \underset{\sim}{E} \qquad \text{Take } \frac{\underset{\sim}{B}\,\cdot}{4\pi} \text{ of this}$$

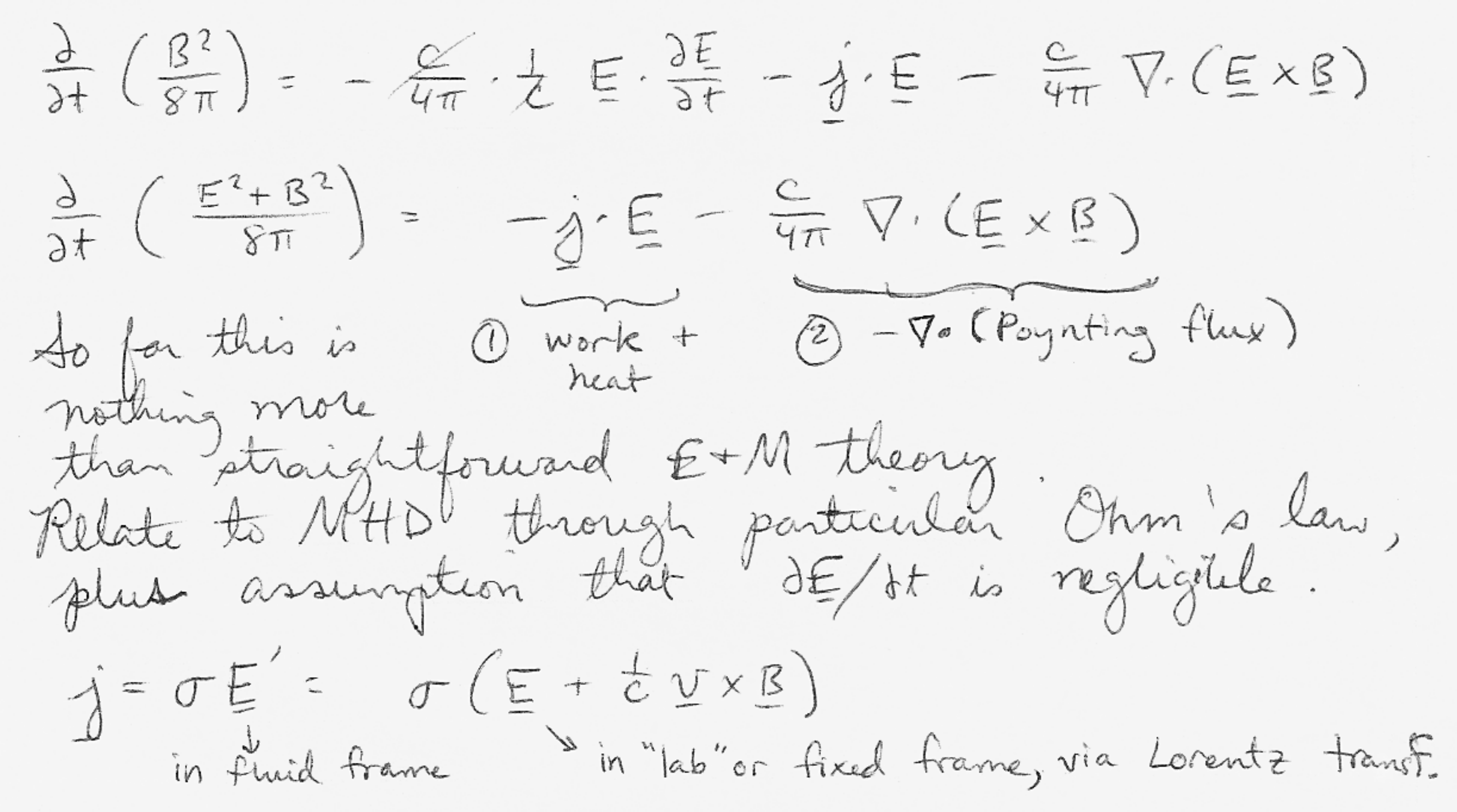
$$\frac{\partial}{\partial t}\left(\frac{B^2}{8\pi}\right) = -\frac{c}{4\pi}\,\underline{B}\cdot\nabla\times\underline{E} \qquad \text{Use} \left\langle \begin{array}{l} \underline{A}\cdot\nabla\times\underline{B} = \\ \underline{B}\cdot\nabla\times\underline{A} - \nabla\cdot(\underline{A}\times\underline{B}) \end{array}\right.$$

$$= -\frac{c}{4\pi}\,\underline{E}\cdot\nabla\times\underline{B} + \frac{c}{4\pi}\,\nabla\cdot(\underset{\sim}{B}\times\underline{E})$$

Ampère's law: $\nabla\times\underline{B} = \frac{4\pi}{c}\,\underline{j} + \frac{1}{c}\frac{\partial E}{\partial t}$

(Note we're assuming $\underline{D} = \underline{E}$ and $\underline{H} = \underline{B}$ throughout.)

MHD Magnetic Energy Equation – 2



$$\frac{\partial}{\partial t}\left(\frac{B^2}{8\pi}\right) = -\frac{\cancel{c}}{4\pi}\cdot\frac{1}{\cancel{c}}\,\underline{E}\cdot\frac{\partial \underline{E}}{\partial t} - \underline{j}\cdot\underline{E} - \frac{c}{4\pi}\nabla\cdot(\underline{E}\times\underline{B})$$

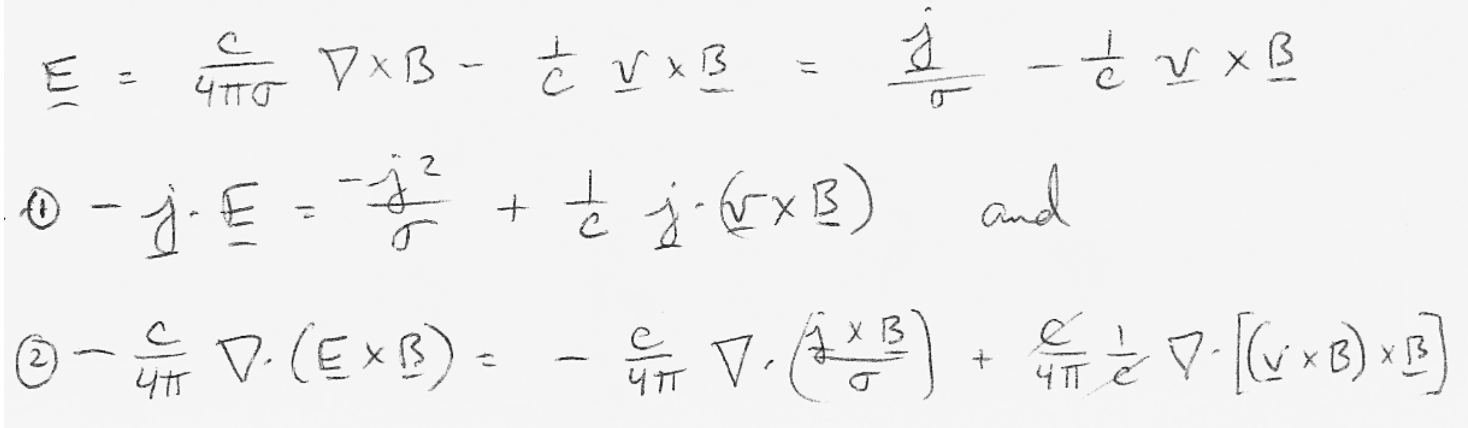
$$\frac{\partial}{\partial t}\left(\frac{E^2+B^2}{8\pi}\right) = \underbrace{-\underline{j}\cdot\underline{E}}_{① \text{ work + heat}} \; \underbrace{-\; \frac{c}{4\pi}\nabla\cdot(\underline{E}\times\underline{B})}_{② \; -\nabla\cdot(\text{Poynting flux})}$$

So far this is nothing more than straightforward E+M theory. Relate to MHD through particular Ohm's law, plus assumption that $\partial \underline{E}/\partial t$ is negligible.

$$\underline{j} = \sigma \underset{\substack{\downarrow \\ \text{in fluid frame}}}{\underline{E}'} = \sigma\left(\underline{E} + \frac{1}{c}\underline{v}\times\underline{B}\right)$$

↘ in "lab" or fixed frame, via Lorentz transf.

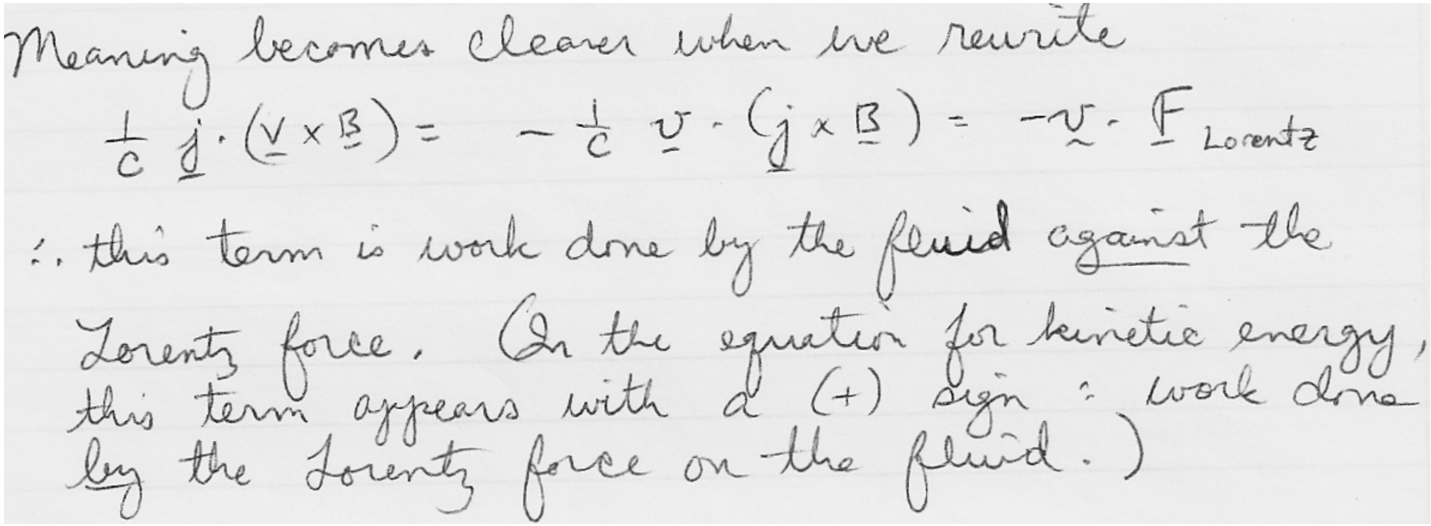
MHD Magnetic Energy Equation – 3



$$\underline{E} = \frac{c}{4\pi\sigma}\nabla\times B - \frac{1}{c}\underline{v}\times\underline{B} = \frac{\underline{j}}{\sigma} - \frac{1}{c}\underline{v}\times\underline{B}$$

① $-\underline{j}\cdot\underline{E} = -\frac{j^2}{\sigma} + \frac{1}{c}\underline{j}\cdot(\underline{v}\times\underline{B})$ and

② $-\frac{c}{4\pi}\nabla\cdot(\underline{E}\times\underline{B}) = -\frac{c}{4\pi}\nabla\cdot\left(\frac{\underline{j}\times\underline{B}}{\sigma}\right) + \frac{\cancel{c}}{4\pi}\frac{1}{\cancel{c}}\nabla\cdot\left[(\underline{v}\times\underline{B})\times\underline{B}\right]$

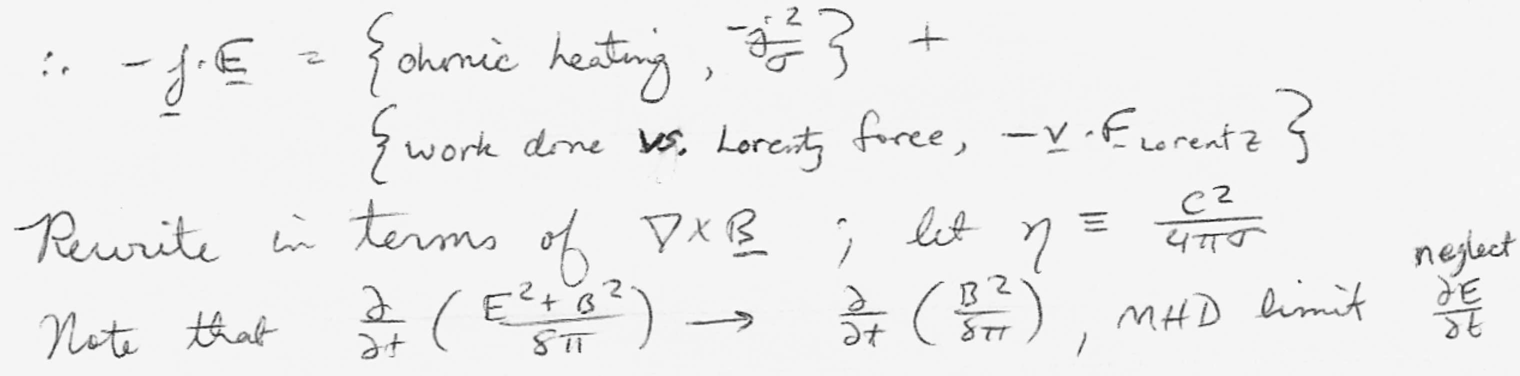


Meaning becomes clearer when we rewrite

$$\frac{1}{c}\underline{j}\cdot(\underline{v}\times\underline{B}) = -\frac{1}{c}\underline{v}\cdot(\underline{j}\times\underline{B}) = -\underset{\sim}{v}\cdot\underline{F}_{Lorentz}$$

∴ this term is work done by the fluid <u>against</u> the Lorentz force. (In the equation for kinetic energy, this term appears with a (+) sign ∴ work done by the Lorentz force on the fluid.)

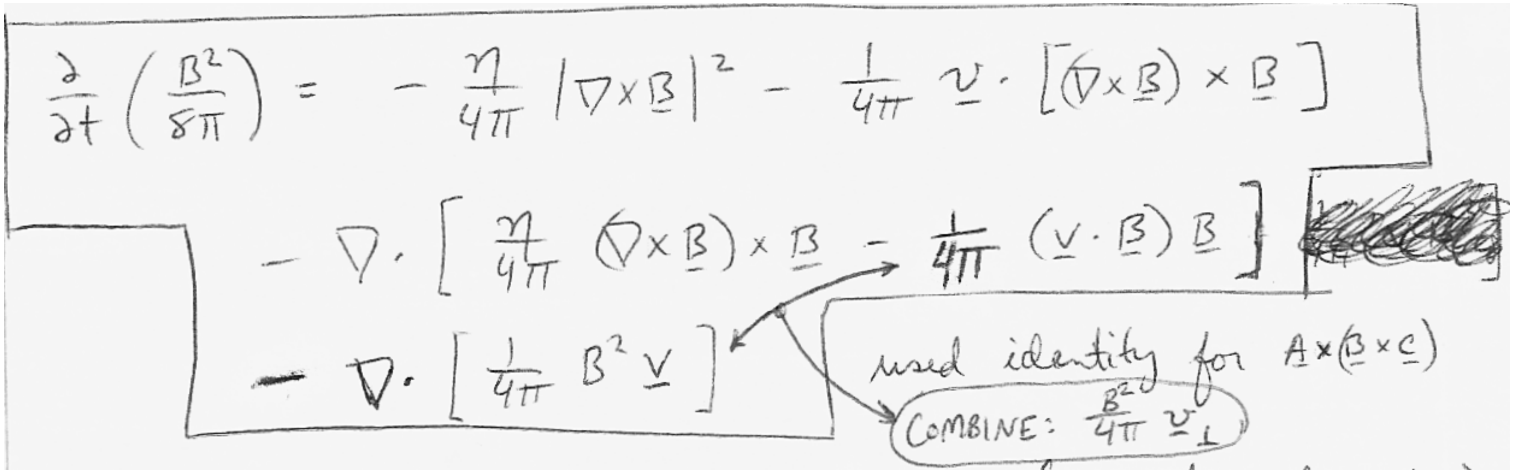
MHD Magnetic Energy Equation – 4



$$\therefore\; -\underline{j}\cdot\underline{E} = \left\{\text{ohmic heating},\ -\frac{j^2}{\sigma}\right\} + \left\{\text{work done vs. Lorentz force},\ -\underline{v}\cdot\underline{F}_{\text{Lorentz}}\right\}$$

Rewrite in terms of $\nabla\times\underline{B}$; let $\eta \equiv \frac{c^2}{4\pi\sigma}$

Note that $\frac{\partial}{\partial t}\left(\frac{E^2+B^2}{8\pi}\right) \longrightarrow \frac{\partial}{\partial t}\left(\frac{B^2}{8\pi}\right)$, MHD limit, neglect $\frac{\partial E}{\partial t}$



$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{B^2}{8\pi}\right) = &-\frac{\eta}{4\pi}\left|\nabla\times\underline{B}\right|^2 - \frac{1}{4\pi}\,\underline{v}\cdot\left[(\nabla\times\underline{B})\times\underline{B}\right] \\ &-\nabla\cdot\left[\frac{\eta}{4\pi}(\nabla\times\underline{B})\times\underline{B} - \frac{1}{4\pi}(\underline{v}\cdot\underline{B})\,\underline{B}\right] \\ &-\nabla\cdot\left[\frac{1}{4\pi}B^2\,\underline{v}\right]\end{aligned}$$

used identity for $\underline{A}\times(\underline{B}\times\underline{C})$

COMBINE: $\frac{B^2}{4\pi}\,\underline{v}_\perp$

MHD Magnetic Energy Equation – 5

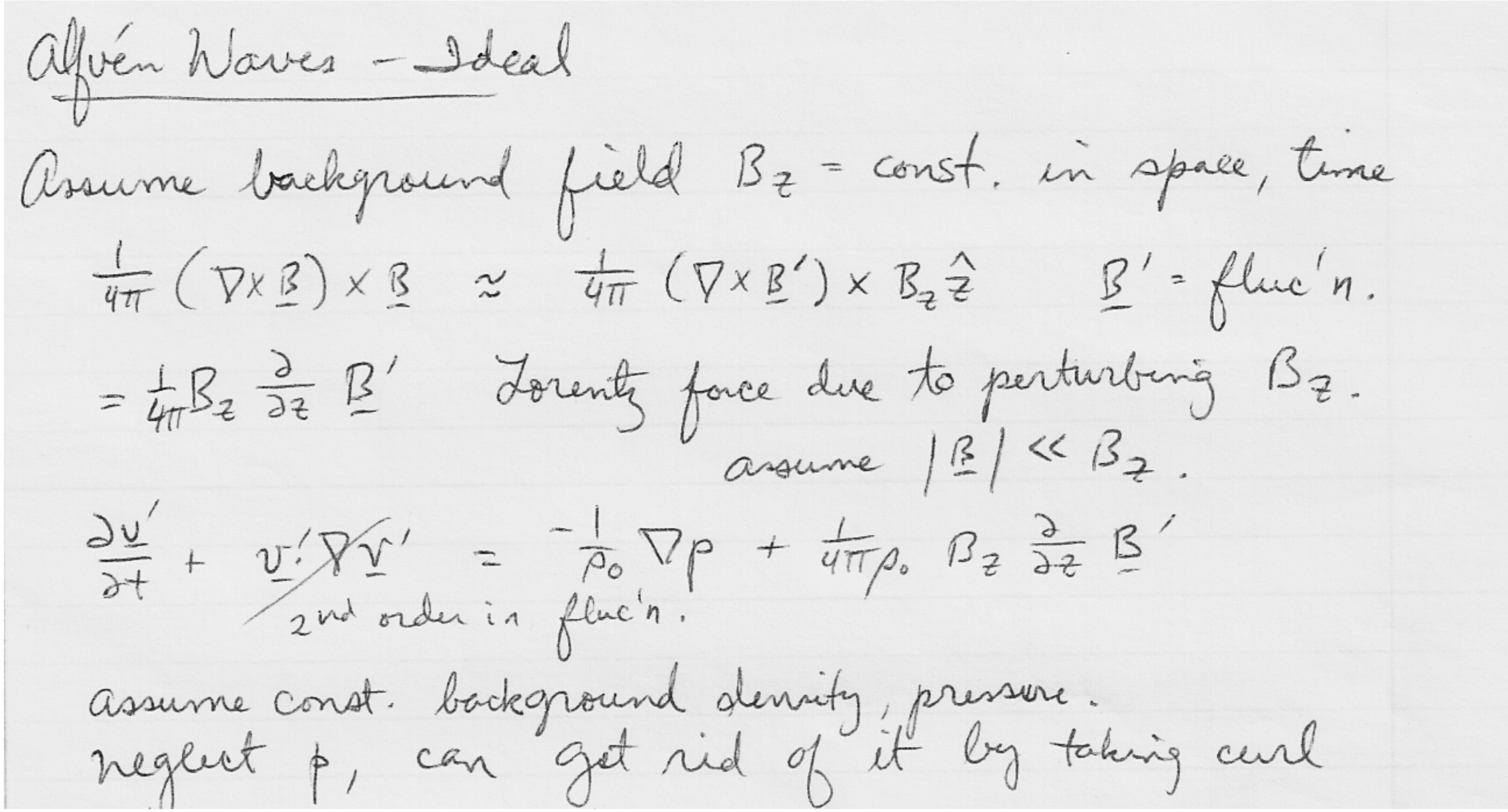
Final form

- After combining and rearranging all terms that involve **v**, the result is:

$$\boxed{\left(\frac{\partial}{\partial t} + \underline{v}\cdot\nabla\right)\frac{B^2}{8\pi} = -\frac{j^2}{\sigma} - \frac{B^2}{4\pi}\left(\nabla_\perp\cdot\underline{v}_\perp\right) - \frac{c}{4\pi}\nabla\cdot\left(\frac{\underline{j}\times\underline{B}}{\sigma}\right)}$$

- Notice that the original Poynting flux due to **v** has been largely cancelled out by terms representing work against the Lorentz force!
- Only two terms with **v** are left:
  - A simple advection term (moved to the left-hand side)
  - A term representing loss of magnetic energy due to sideways spreading of flux
- *The only real energy sink is ohmic heating,* j^2/σ
- One can equally well derive this result from the MHD induction law

Alfvén Waves in MHD – 1



Alfvén Waves – Ideal

Assume background field B_z = const. in space, time

$$\frac{1}{4\pi}(\nabla\times\underline{B})\times\underline{B} \simeq \frac{1}{4\pi}(\nabla\times\underline{B}')\times B_z\hat{z} \qquad \underline{B}' = \text{fluc'n.}$$

$$= \frac{1}{4\pi}B_z\frac{\partial}{\partial z}\underline{B}'$$

Lorentz force due to perturbing B_z.

assume $|\underline{B}| \ll B_z$.

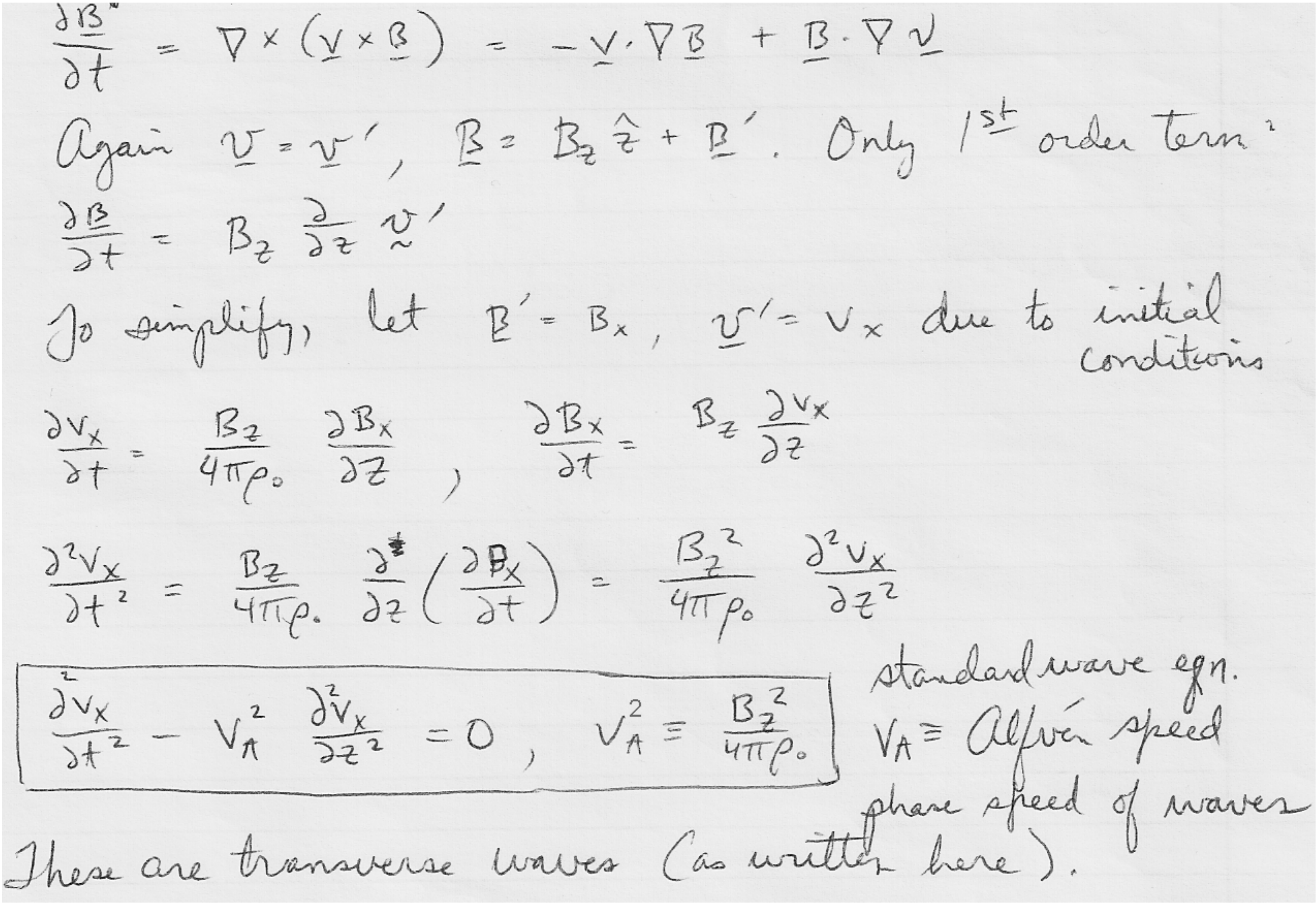
$$\frac{\partial \underline{v}'}{\partial t} + \cancel{\underline{v}'\cdot\nabla\underline{v}'} = -\frac{1}{\rho_0}\nabla p + \frac{1}{4\pi\rho_0}B_z\frac{\partial}{\partial z}\underline{B}'$$

2nd order in fluc'n.

assume const. background density, pressure.

neglect p, can get rid of it by taking curl

Alfvén Waves in MHD – 2



$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{v} \times \underline{B}) = -\underline{v} \cdot \nabla \underline{B} + \underline{B} \cdot \nabla \underline{v}$$

Again $\underline{v} = \underline{v}'$, $\underline{B} = B_z \hat{z} + \underline{B}'$. Only 1st order term:

$$\frac{\partial \underline{B}}{\partial t} = B_z \frac{\partial}{\partial z} \underline{v}'$$

To simplify, let $\underline{B}' = B_x$, $\underline{v}' = v_x$ due to initial conditions

$$\frac{\partial v_x}{\partial t} = \frac{B_z}{4\pi\rho_0} \frac{\partial B_x}{\partial z}, \qquad \frac{\partial B_x}{\partial t} = B_z \frac{\partial v_x}{\partial z}$$

$$\frac{\partial^2 v_x}{\partial t^2} = \frac{B_z}{4\pi\rho_0} \frac{\partial}{\partial z}\left(\frac{\partial B_x}{\partial t}\right) = \frac{B_z^2}{4\pi\rho_0} \frac{\partial^2 v_x}{\partial z^2}$$

$$\boxed{\frac{\partial^2 v_x}{\partial t^2} - V_A^2 \frac{\partial^2 v_x}{\partial z^2} = 0, \quad V_A^2 \equiv \frac{B_z^2}{4\pi\rho_0}}$$

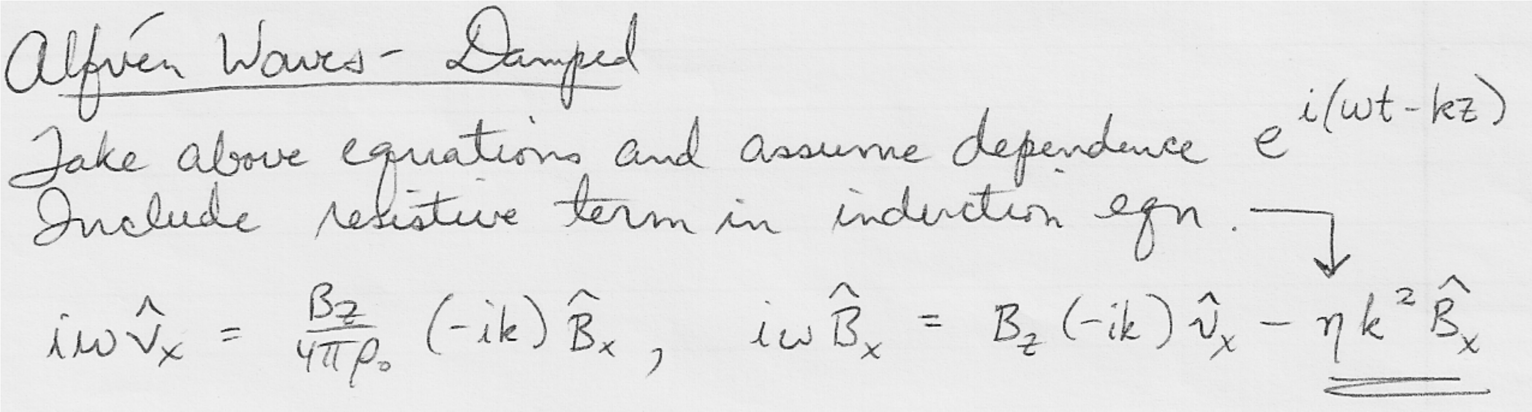
standard wave eqn.

$V_A \equiv$ Alfvén speed

phase speed of waves

These are transverse waves (as written here).

Alfvén Waves in MHD – 3

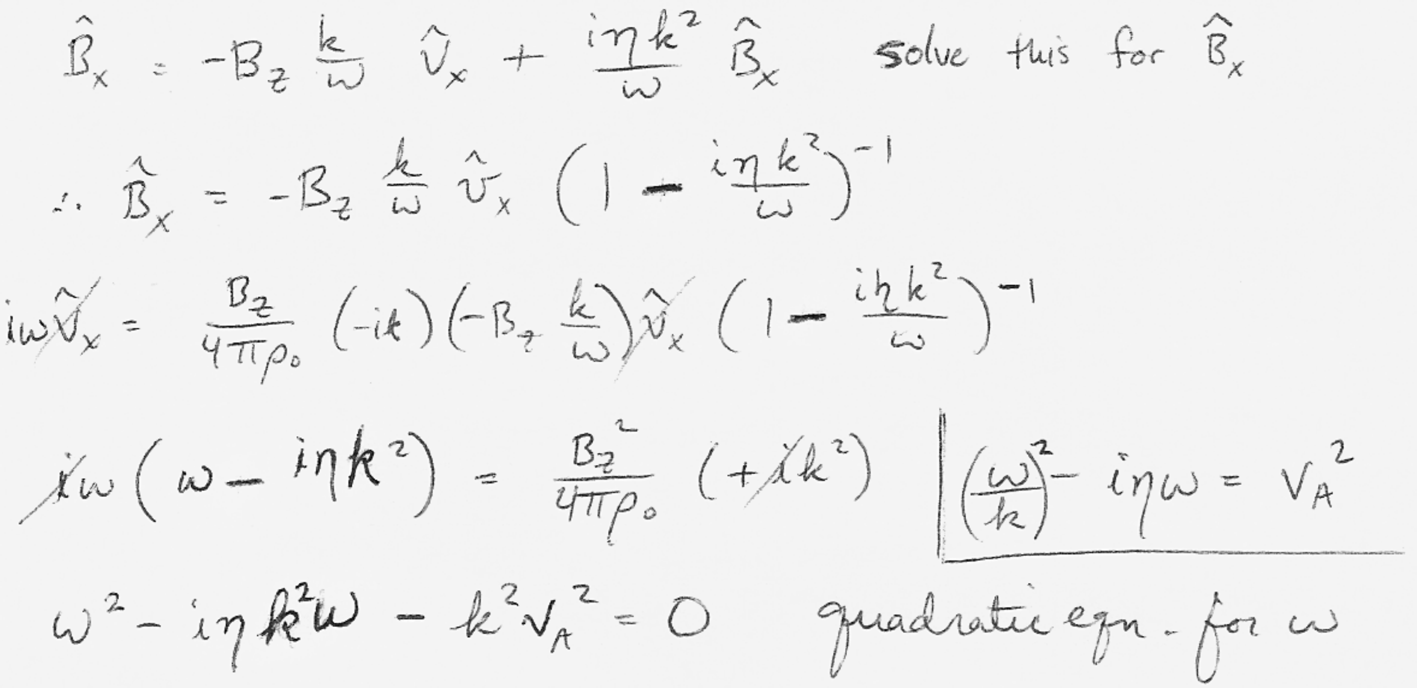


Alfvén Waves - Damped

Take above equations and assume dependence $e^{i(\omega t - kz)}$

Include resistive term in induction eqn.

$$i\omega \hat{v}_x = \frac{B_z}{4\pi\rho_0}(-ik)\hat{B}_x , \qquad i\omega \hat{B}_x = B_z(-ik)\hat{v}_x - \underline{\underline{\eta k^2 \hat{B}_x}}$$



$\hat{B}_x = -B_z \frac{k}{\omega} \hat{v}_x + \frac{i\eta k^2}{\omega} \hat{B}_x$ solve this for $\hat{B}_x$

$$\therefore \hat{B}_x = -B_z \frac{k}{\omega} \hat{v}_x \left(1 - \frac{i\eta k^2}{\omega}\right)^{-1}$$

$$i\omega \cancel{\hat{v}_x} = \frac{B_z}{4\pi\rho_0}(-ik)\left(-B_z\frac{k}{\omega}\right)\cancel{\hat{v}_x}\left(1 - \frac{i\eta k^2}{\omega}\right)^{-1}$$

$$\cancel{i}\omega(\omega - i\eta k^2) = \frac{B_z^2}{4\pi\rho_0}(+\cancel{i}k^2) \qquad \left(\frac{\omega}{k}\right)^2 - i\eta\omega = V_A^2$$

$\omega^2 - i\eta k^2\omega - k^2 V_A^2 = 0$ quadratic eqn. for ω

Alfvén Waves in MHD – 4

$$\omega = \frac{i\eta k^2 \pm \sqrt{-\eta^2 k^4 + 4k^2 v_A^2}}{2} \qquad \eta = 0 \Rightarrow \omega = \pm k v_A \checkmark$$

$$\boxed{\omega = k v_A \left[\frac{i\eta k}{2 v_A} \pm \sqrt{1 - \frac{\eta^2 k^2}{4 v_A^2}} \right]}$$

Real part: $\omega_r = \pm k v_A \sqrt{1 - \frac{\eta^2 k^2}{4 v_A^2}}$

Imag part: $\omega_i = k v_A \left(\frac{i\eta k}{2 v_A} \right) \sim \omega_r (Lu)^{-1}$

$Lu \equiv$ Lundquist no. $= \frac{L v_A}{\eta}$: very big in solar corona

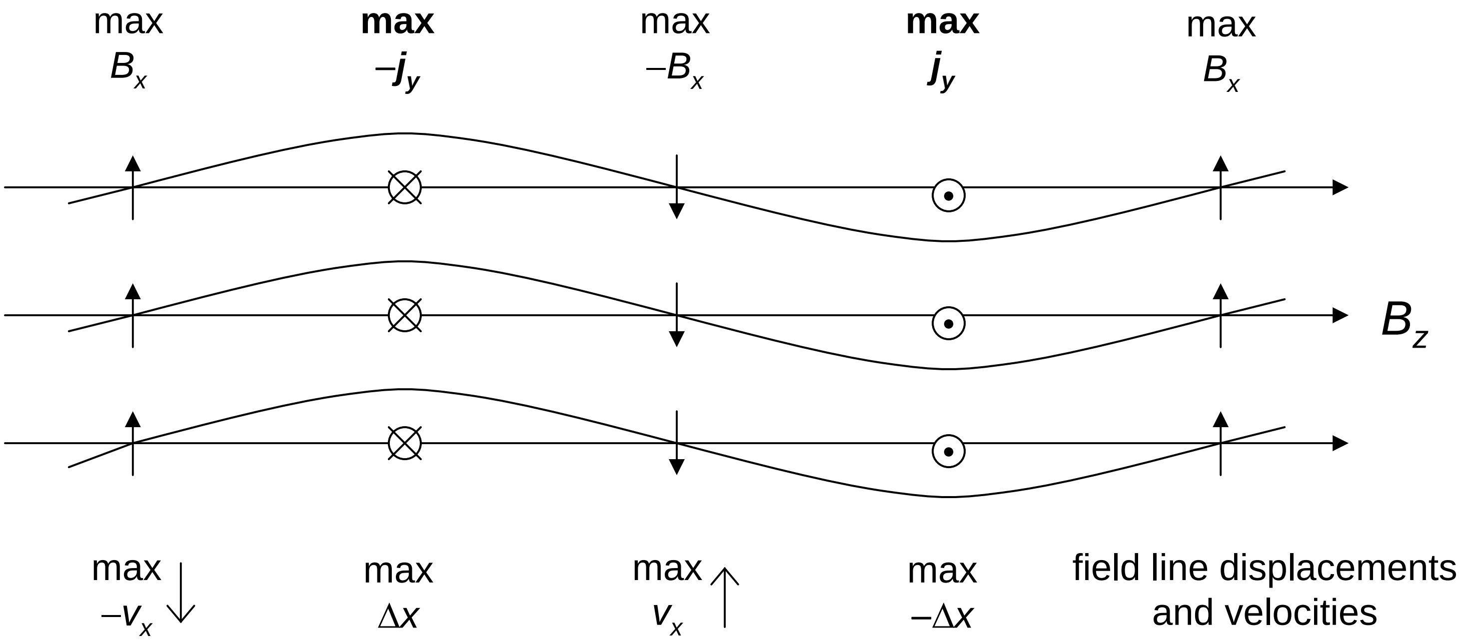
$\therefore \left(\frac{i\eta k}{2 v_A} \right)^2$ is a small correction to $\omega_r \simeq \pm k v_A \left[1 - \frac{1}{2} \left(\frac{\eta k}{2 v_A} \right)^2 \right]$

Note $e^{i\omega_i t} = \exp\left(i \left(\frac{i\eta k^2}{2} \right) t \right) = \exp\left(- \underbrace{\frac{\eta k^2}{2} t}_{\sim (Lu)^{-1} \omega t\ !} \right)$

exponentially damped, as expected.

Why Are the Alfvén Waves Damped?

- Ideal wave (below) depends on transverse, “frozen-in” displacements of B_z
- Resistivity weakens the necessary currents, causing the amplitude to slip



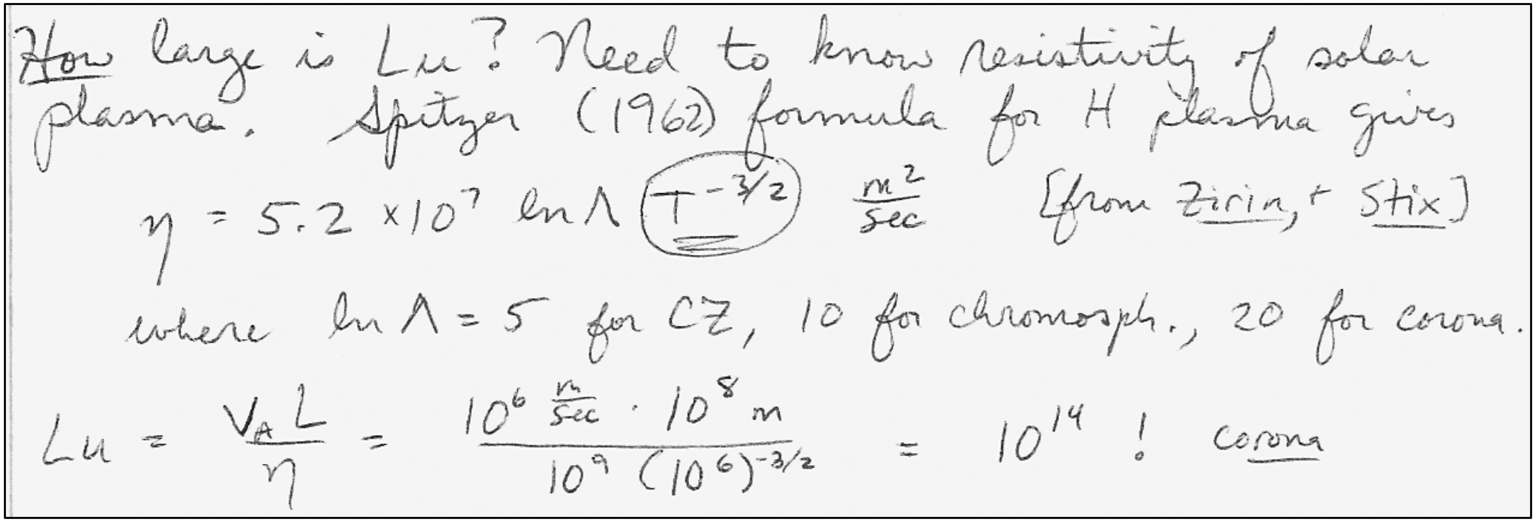
1) Ideal right-traveling wave goes like exp($i\omega t - ikz$) $\rightarrow$ $j_y = -ikB_x(c/4\pi)$

2) Using $\omega = kv_A$: $i\omega v_x = j_y B_z/(c\rho_0)$, $ikv_A v_x = -ikB_x(v_A{}^2/B_z)$, $\boxed{v_x/v_A = -B_x/B_z}$

3) Integrate over *dt* to show that fluid and field line displacements are equal

The Lundquist Number in the Solar Corona

- The Lundquist number is the dimensionless ratio of two timescales:
  - Alfvén wave travel time over a distance L
  - Resistive diffusion time over the same distance
- It is equal to the magnetic Reynolds number divided by the Alfvén Mach number, R_m / M_A



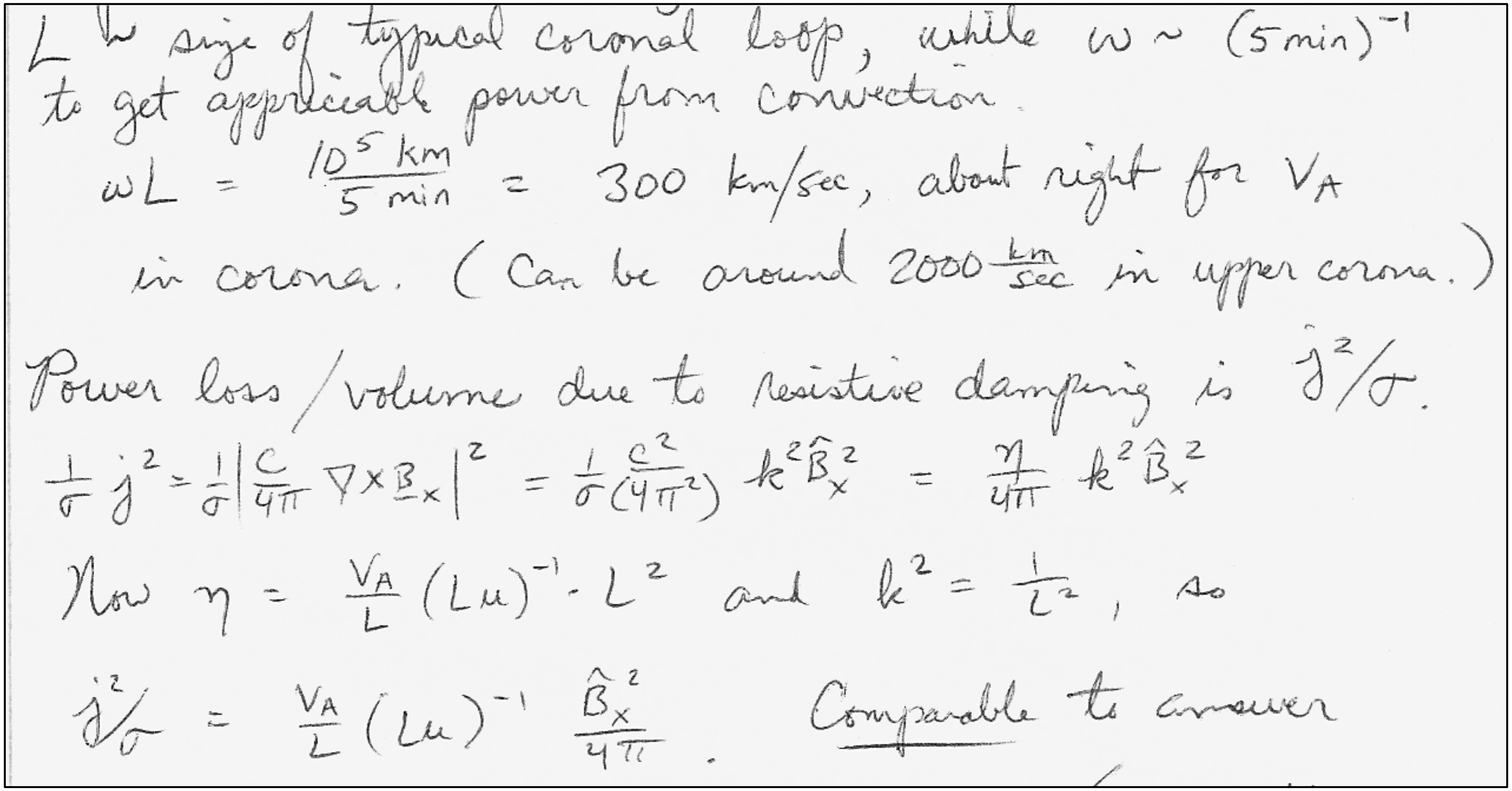
How large is Lu? Need to know resistivity of solar plasma. Spitzer (1962) formula for H plasma gives

$$\eta = 5.2 \times 10^7 \ln \Lambda \boxed{\underline{T}^{-3/2}} \frac{m^2}{sec} \quad \text{[from Zirin, + Stix]}$$

where $\ln \Lambda = 5$ for CZ, 10 for chromosph., 20 for corona.

$$Lu = \frac{V_A L}{\eta} = \frac{10^6 \frac{m}{sec} \cdot 10^8 m}{10^9 (10^6)^{-3/2}} = 10^{14} \text{ ! corona}$$

Estimate of Heating Rate Due to Alfvén Wave Damping



L the size of typical coronal loop, while $\omega \sim (5\,\text{min})^{-1}$ to get appreciable power from convection.

$$\omega L = \frac{10^5\,\text{km}}{5\,\text{min}} = 300\ \text{km/sec},$$ about right for V_A in corona. (Can be around $2000\ \frac{\text{km}}{\text{sec}}$ in upper corona.)

Power loss / volume due to resistive damping is j^2/σ.

$$\frac{1}{\sigma} j^2 = \frac{1}{\sigma}\left|\frac{c}{4\pi}\nabla\times\underline{B}_x\right|^2 = \frac{1}{\sigma}\frac{c^2}{(4\pi^2)}k^2\hat{B}_x^2 = \frac{\eta}{4\pi}k^2\hat{B}_x^2$$

Now $\eta = \frac{V_A}{L}(Lu)^{-1}\cdot L^2$ and $k^2 = \frac{1}{L^2}$, so

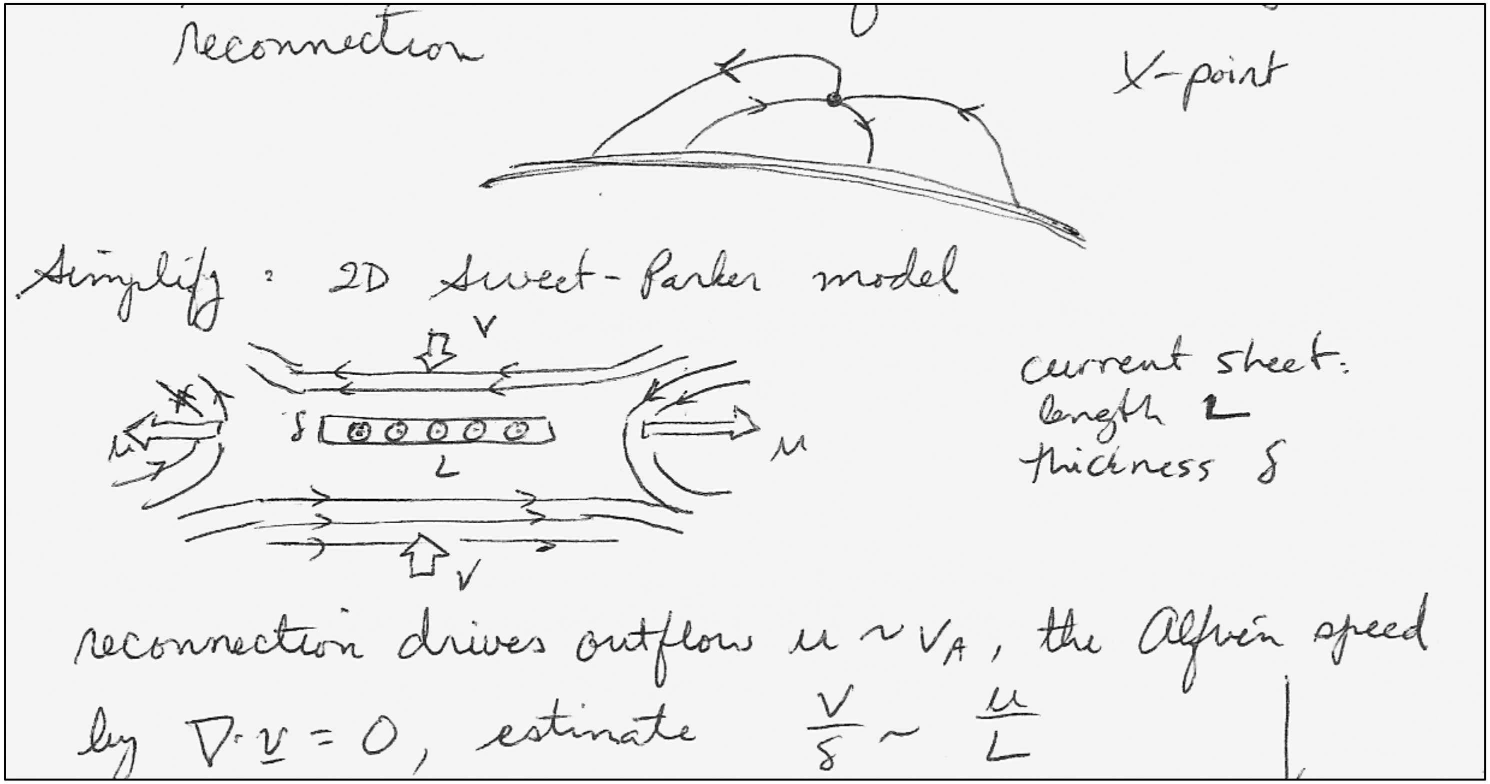
$$j^2/\sigma = \frac{V_A}{L}(Lu)^{-1}\frac{\hat{B}_x^2}{4\pi}.$$ Comparable to answer

Exponentially decaying wave is identical: $\partial/\partial t\,\left|B_x^2/(8\pi)\right| = 2\omega_i\left|B_x^2/(8\pi)\right|$

…How does this stack up against the nanoflare/reconnection model?

Sweet-Parker Model of Reconnection – 1

It’s only a 2D model, but it takes into account that the reconnection region must be very thin when the diffusivity is extremely low



Reconnection

Simplify: 2D Sweet-Parker model

current sheet:
length L
thickness δ

reconnection drives outflow $u \sim V_A$, the Alfvén speed

by $\nabla \cdot \underline{v} = 0$, estimate $\frac{V}{\delta} \sim \frac{u}{L}$

How big can the inflow be, given these geometric constraints?

Sweet-Parker Model of Reconnection – 2

We estimate the layer thickness from MHD magnetic induction, and the outflow speed by assuming it is driven by the Lorentz force

Finally, need to balance inflow rate with diffusion (reconnection)

$$\frac{vB}{\delta} \sim \frac{\eta B}{\delta^2} \Rightarrow \delta \sim \frac{\eta}{v}$$

because magnetic pressure drives flow

$$\frac{B^2}{8\pi} \sim \frac{1}{2}\rho v^2$$

$$u \sim \frac{B}{\sqrt{4\pi\rho}} \equiv V_A$$

$$\therefore\ v \sim \frac{\delta}{L} V_A \sim \frac{\eta}{vL} V_A$$

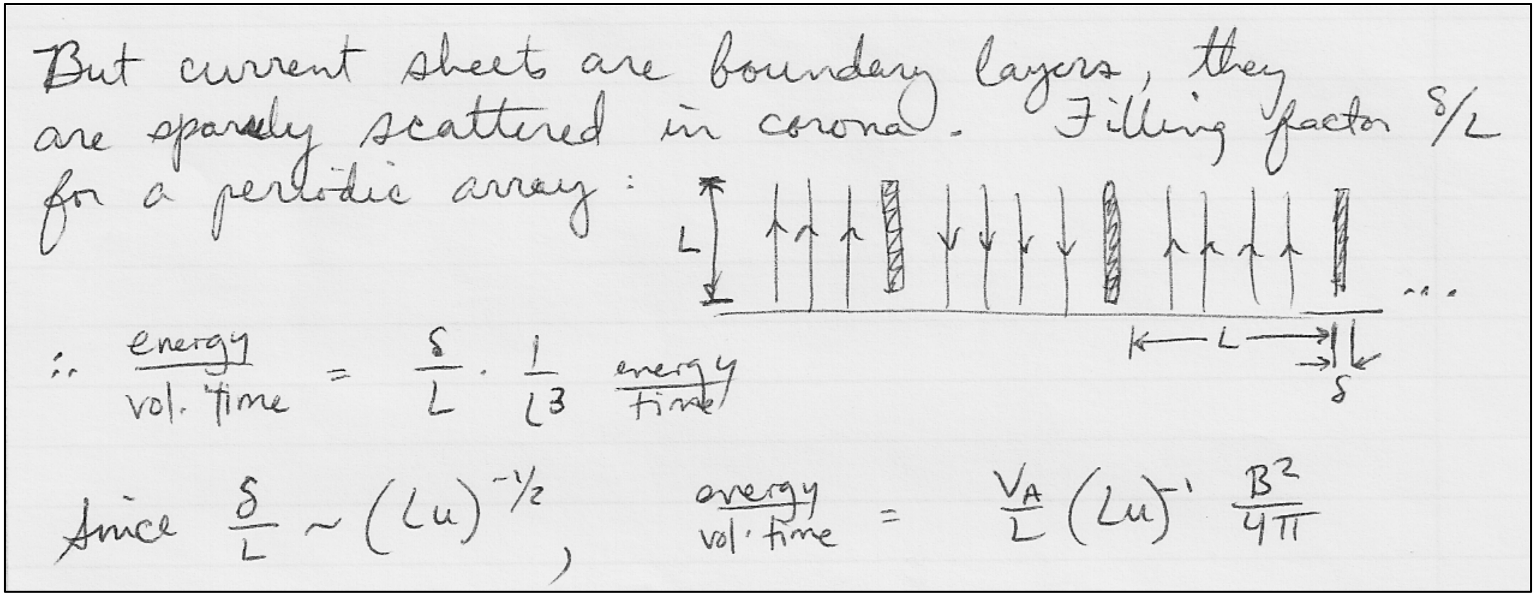
$$\Rightarrow v \sim \left(\frac{\eta V_A}{L}\right)^{1/2} = V_A \left(\frac{\eta}{V_A L}\right)^{1/2} = V_A (Lu)^{-1/2}$$

where $Lu \equiv$ Lundquist number $\equiv \frac{V_A L}{\eta}$

Once again, the Lundquist number comes into play…

Sweet-Parker Model of Reconnection – 3

Unfortunately, the low rate of magnetic energy conversion is reduced even further if *L* also approximates distance *between* current sheets:



But current sheets are boundary layers, they are sparsely scattered in corona. Filling factor δ/L for a periodic array:

$$\therefore \frac{\text{energy}}{\text{vol.}\cdot\text{time}} = \frac{\delta}{L}\cdot\frac{1}{L^3}\,\frac{\text{energy}}{\text{time}}$$

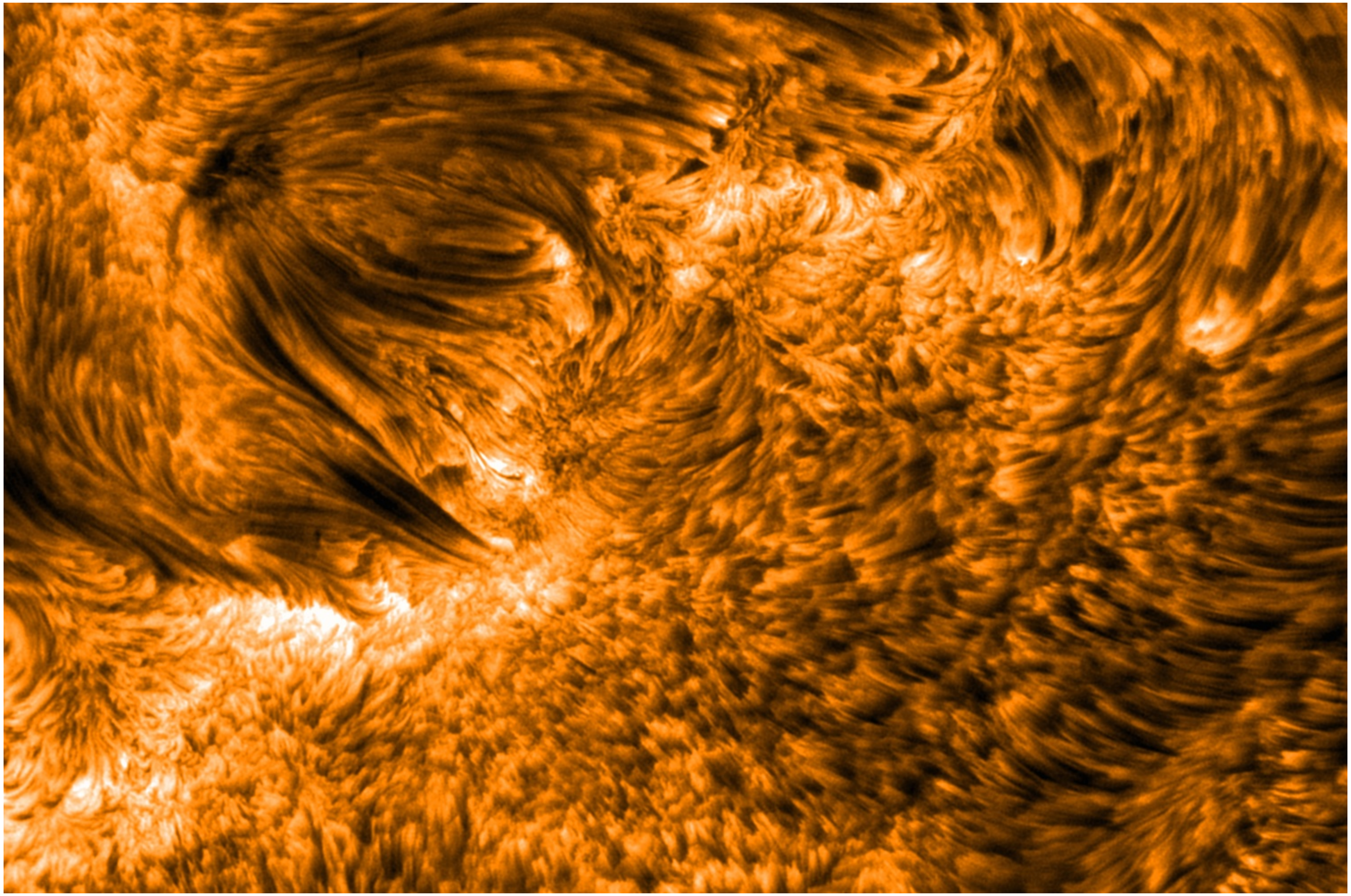
Since $\frac{\delta}{L} \sim (Lu)^{-1/2}$, $\frac{\text{energy}}{\text{vol}\cdot\text{time}} = \frac{v_A}{L}(Lu)^{-1}\frac{B^2}{4\pi}$

It is possible to improve the $(Lu)^{-1/2}$ to ln(Lu) through better models, such as the ones by Petschek or by Sonnerup and Priest, which have refinements:

- The plasma is compressible—fast or slow magnetosonic shocks allow $u > v_A$
- The incoming magnetic field is bent by shocks, so outflow is broader (in 2D)

Spicules/Fibrils (on the limb/disk)

A possible effect of sound waves on the solar atmosphere

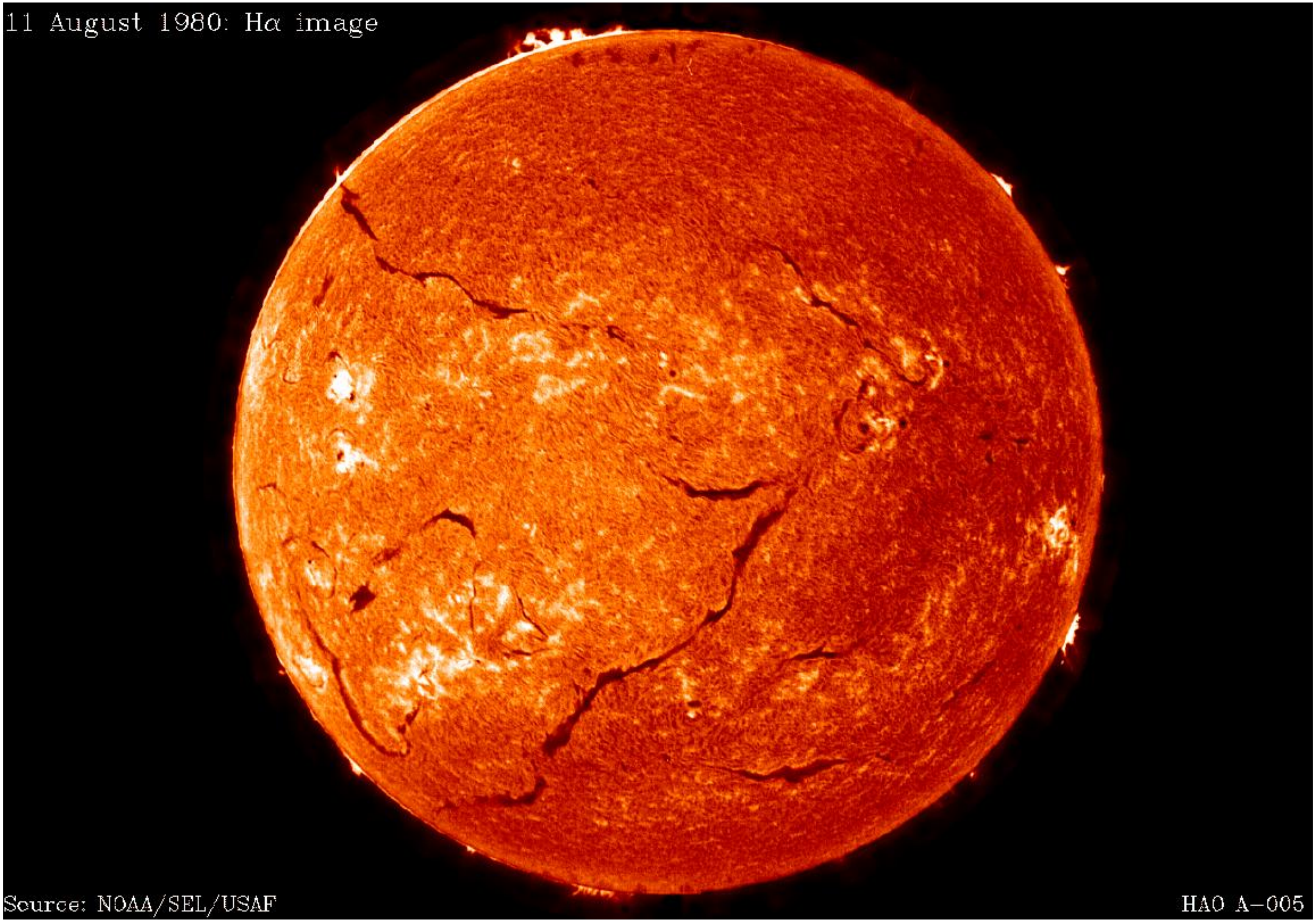


Short-lived, tall jets in the Hα chromosphere may be driven by *p*-modes

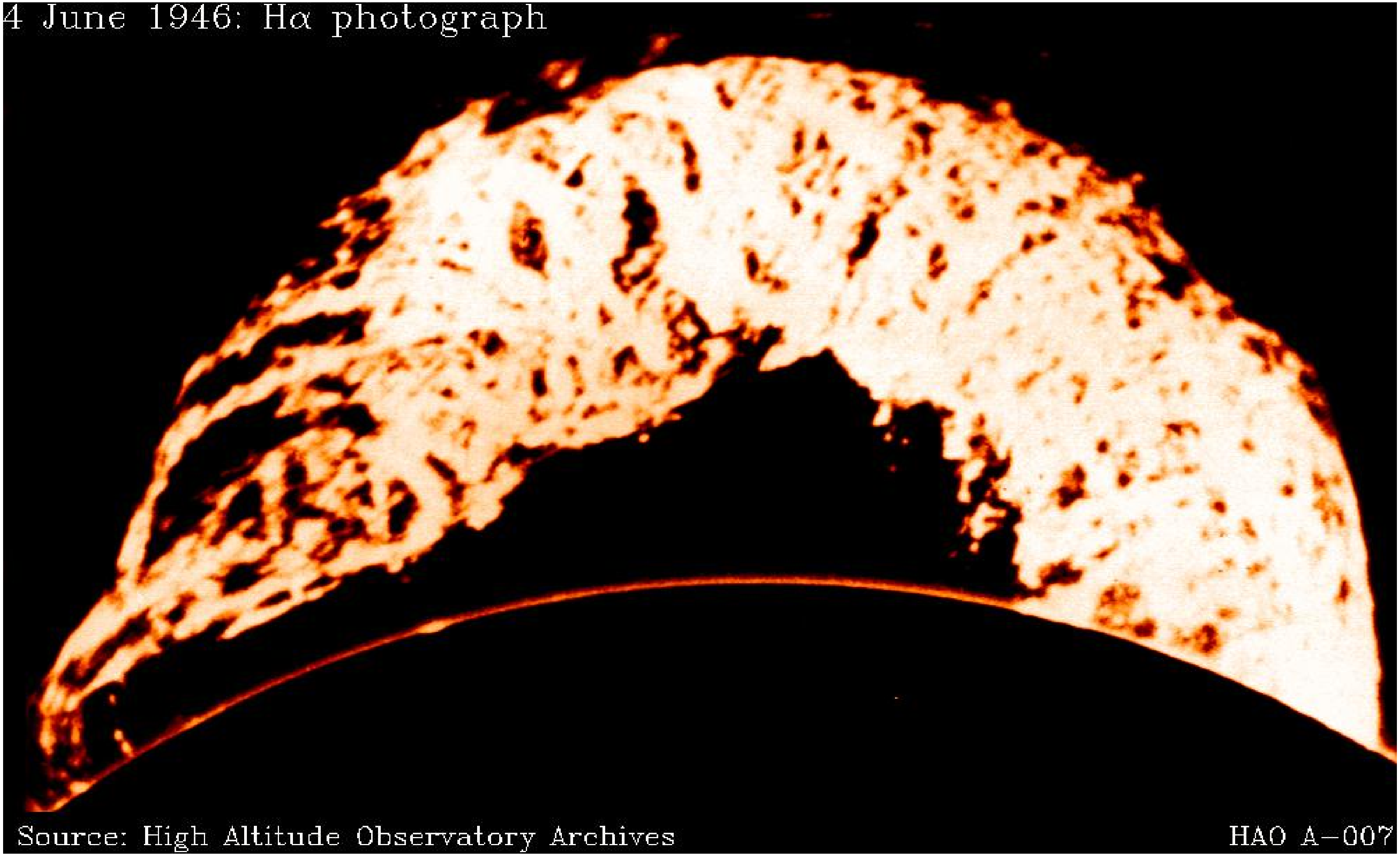
Credit: Royal Swedish Academy of Sciences
**See QuickTime movie of spicules in action*

Filaments and Prominences Viewed in Hα

They are condensations of cooler gas suspended in the corona

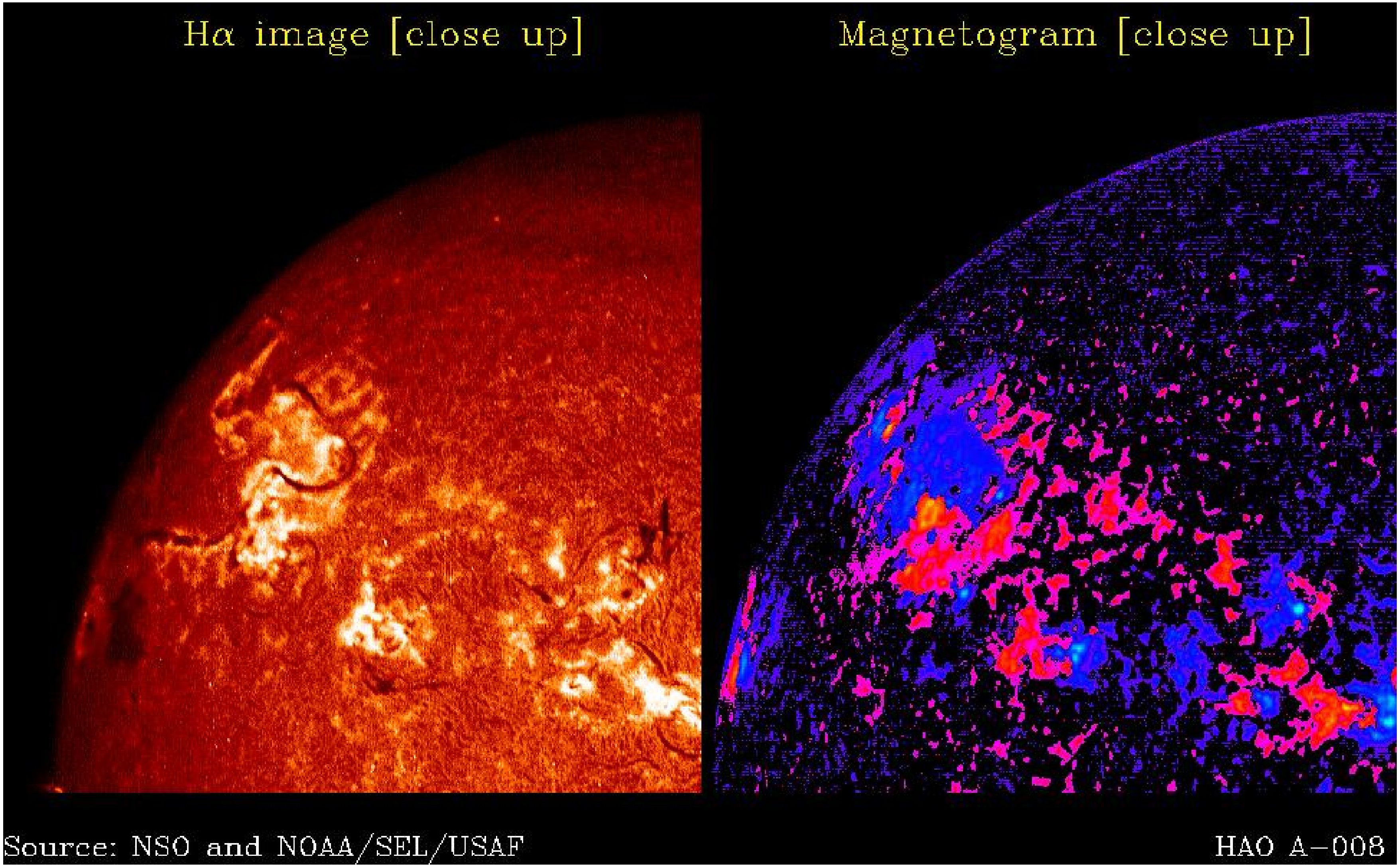


Prominences Can Be Very… Prominent!

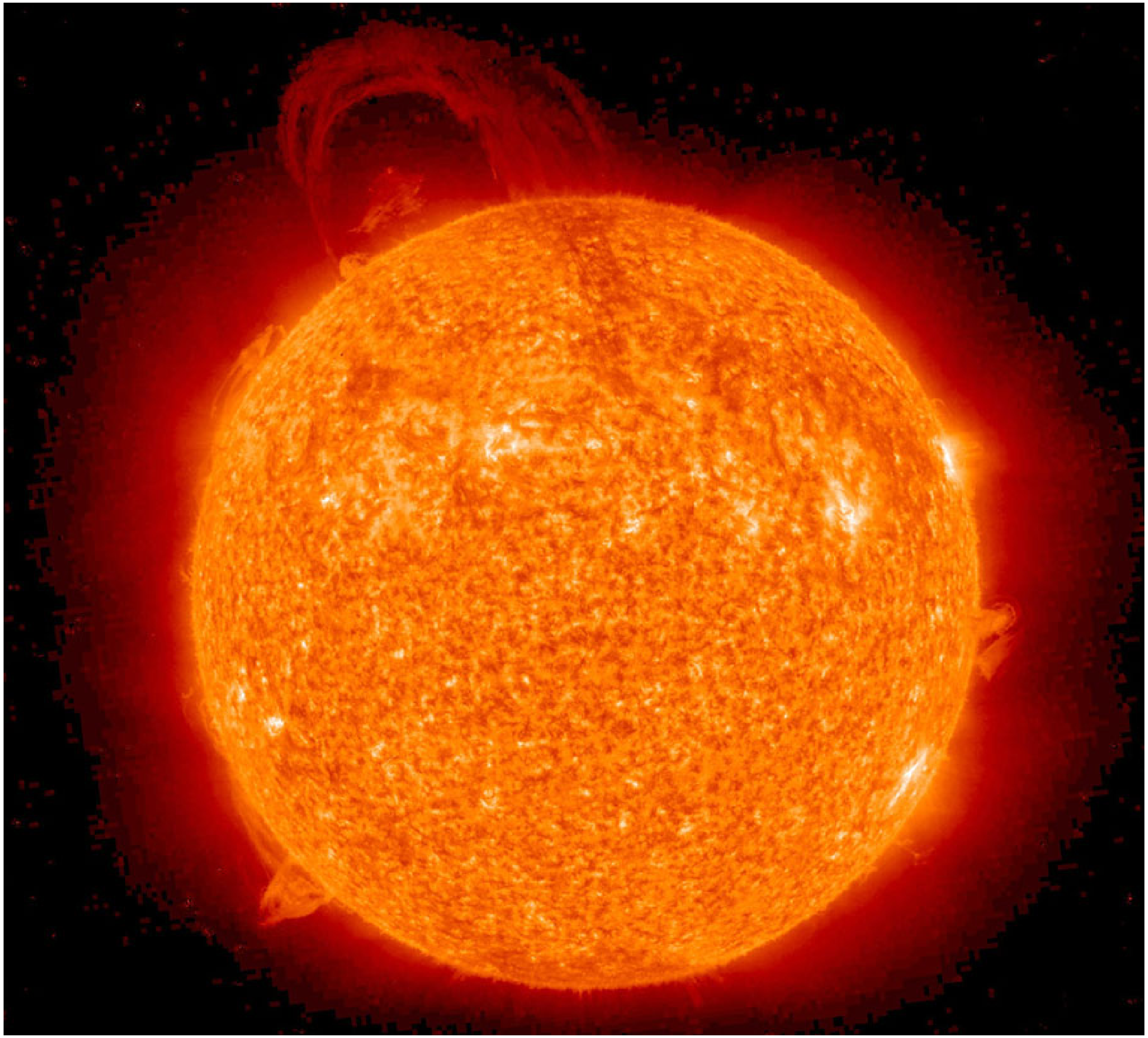


Filaments Tend to Form on Magnetic Neutral Lines

This gives us a clue about what holds them up

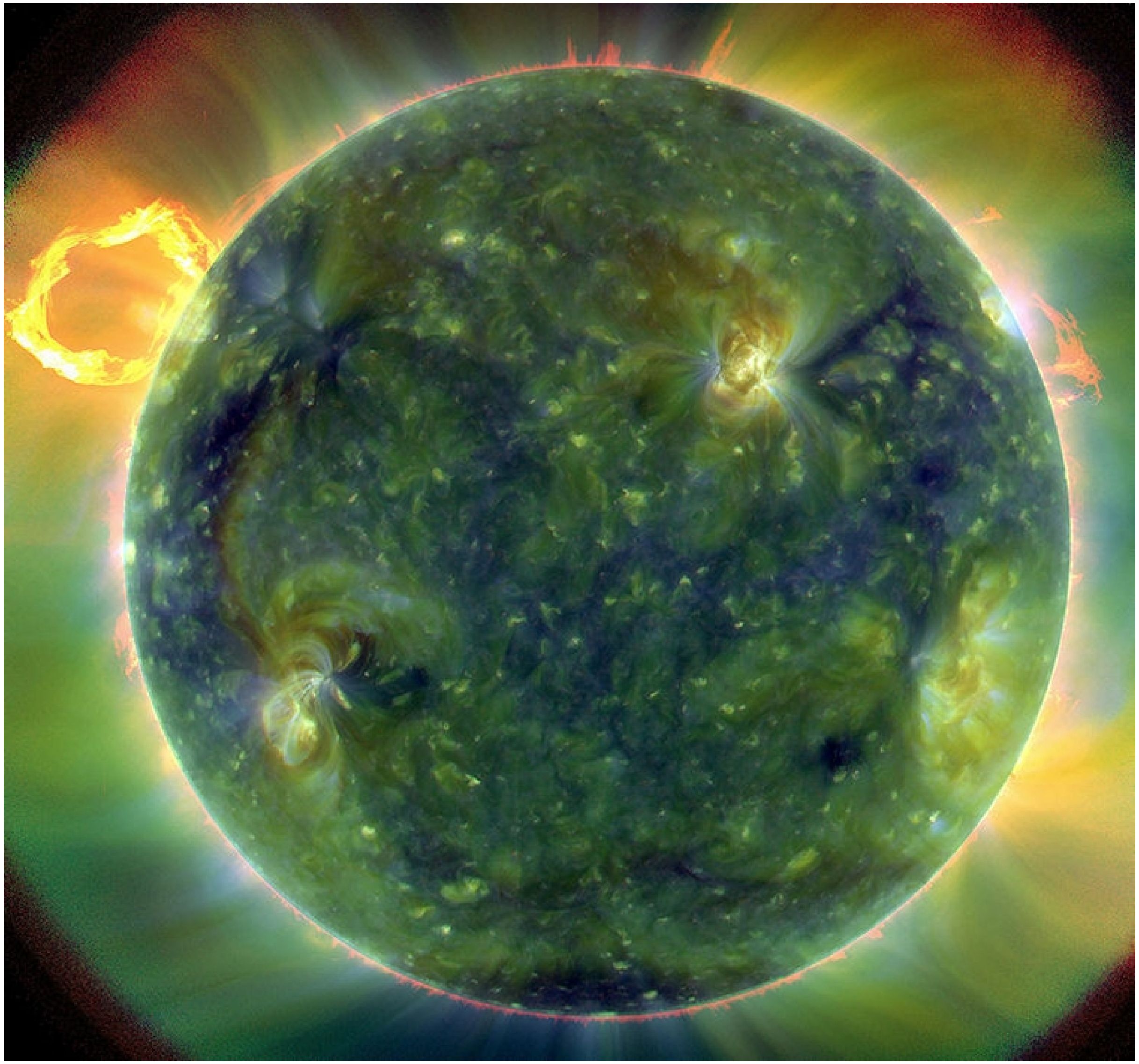


Huge Eruptive Prominence Captured by STEREO



**QuickTime movie shows all the action*

Eruptive Prominence from SDO First Light



http://science.nasa.gov/science-news/science-at-nasa/2010/21apr_firstlight/

Zoomed-In Animation of Eruptive Prominence

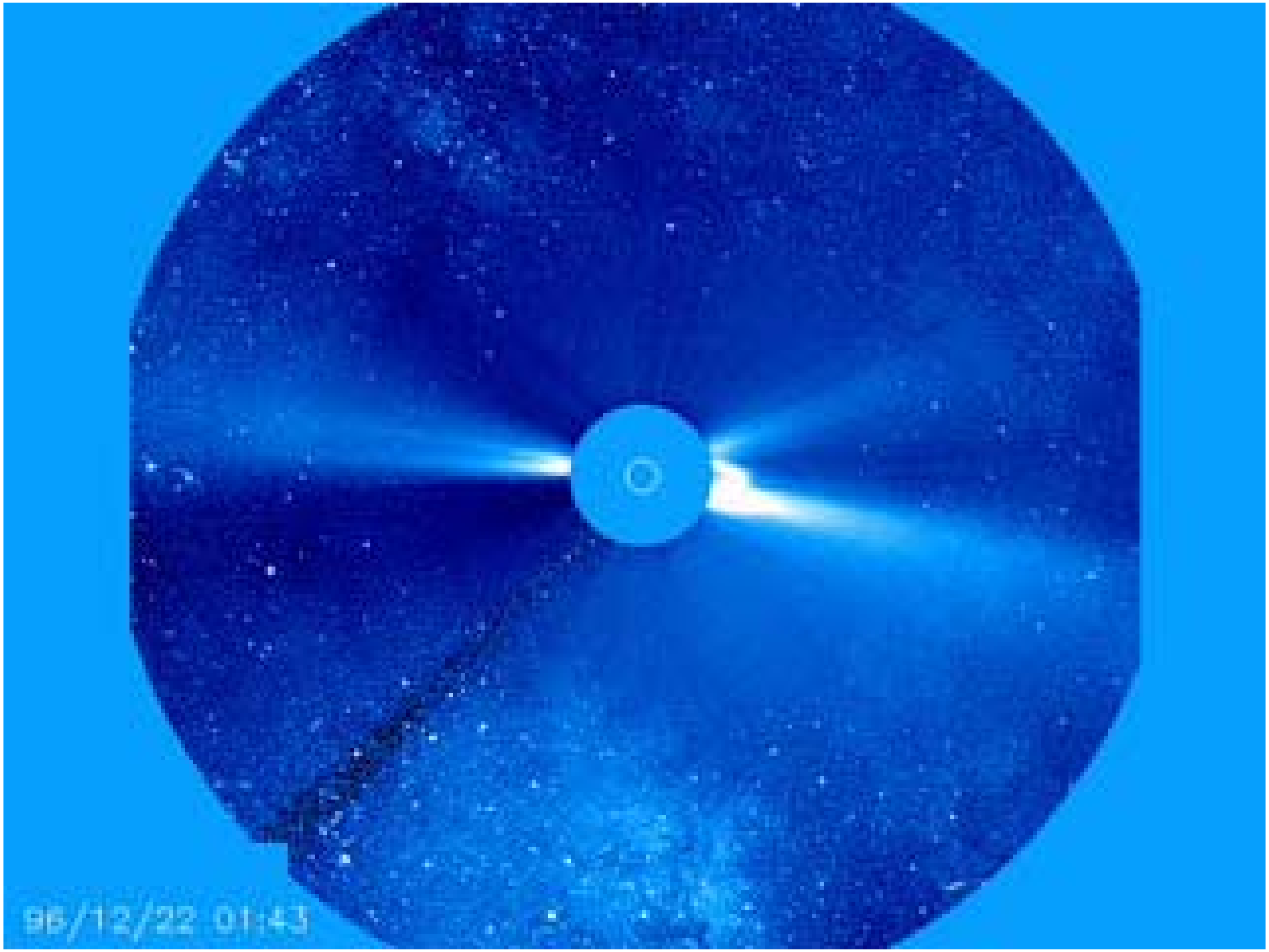
Watch for the twist in the plumes of plasma as they descend

NASA/GSFC/SVS/SDO

**QuickTime movie shows the event*

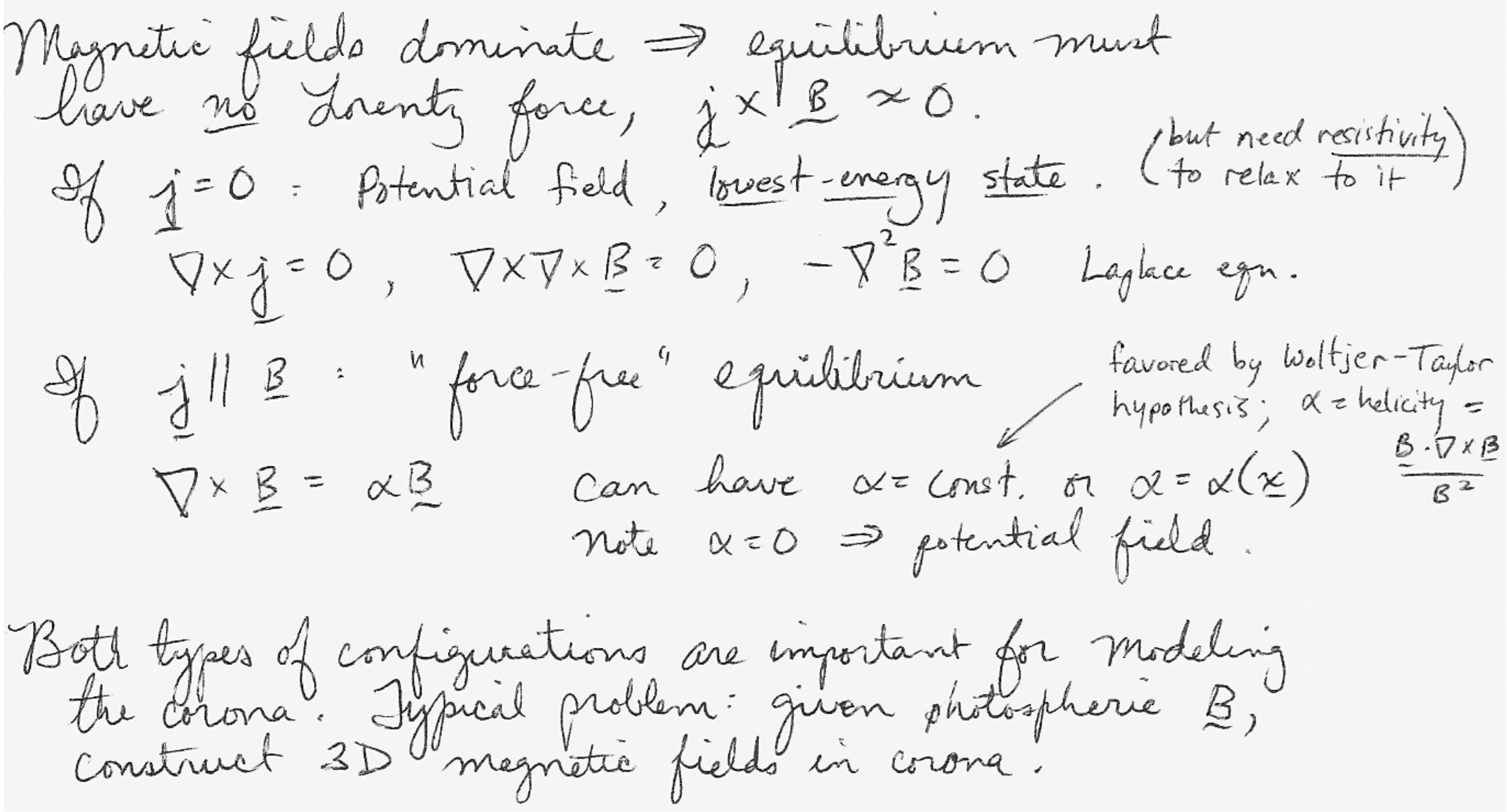
A Coronal Mass Ejection Witnessed by SOHO/LASCO

CME events are often associated with eruptive prominences



Coronal Structures – 1

Possible MHD equilibria for long-lived formations



Magnetic fields dominate ⇒ equilibrium must have no Lorentz force, $\underset{\sim}{j} \times \underset{\sim}{B} \approx 0$.

If $\underset{\sim}{j} = 0$: Potential field, lowest-energy state. (but need resistivity to relax to it)

$\nabla \times \underset{\sim}{j} = 0$, $\nabla \times \nabla \times \underset{\sim}{B} = 0$, $-\nabla^2 \underset{\sim}{B} = 0$ Laplace eqn.

If $\underset{\sim}{j} \parallel \underset{\sim}{B}$: "force-free" equilibrium

favored by Woltjer-Taylor hypothesis; α = helicity = $\frac{B \cdot \nabla \times B}{B^2}$

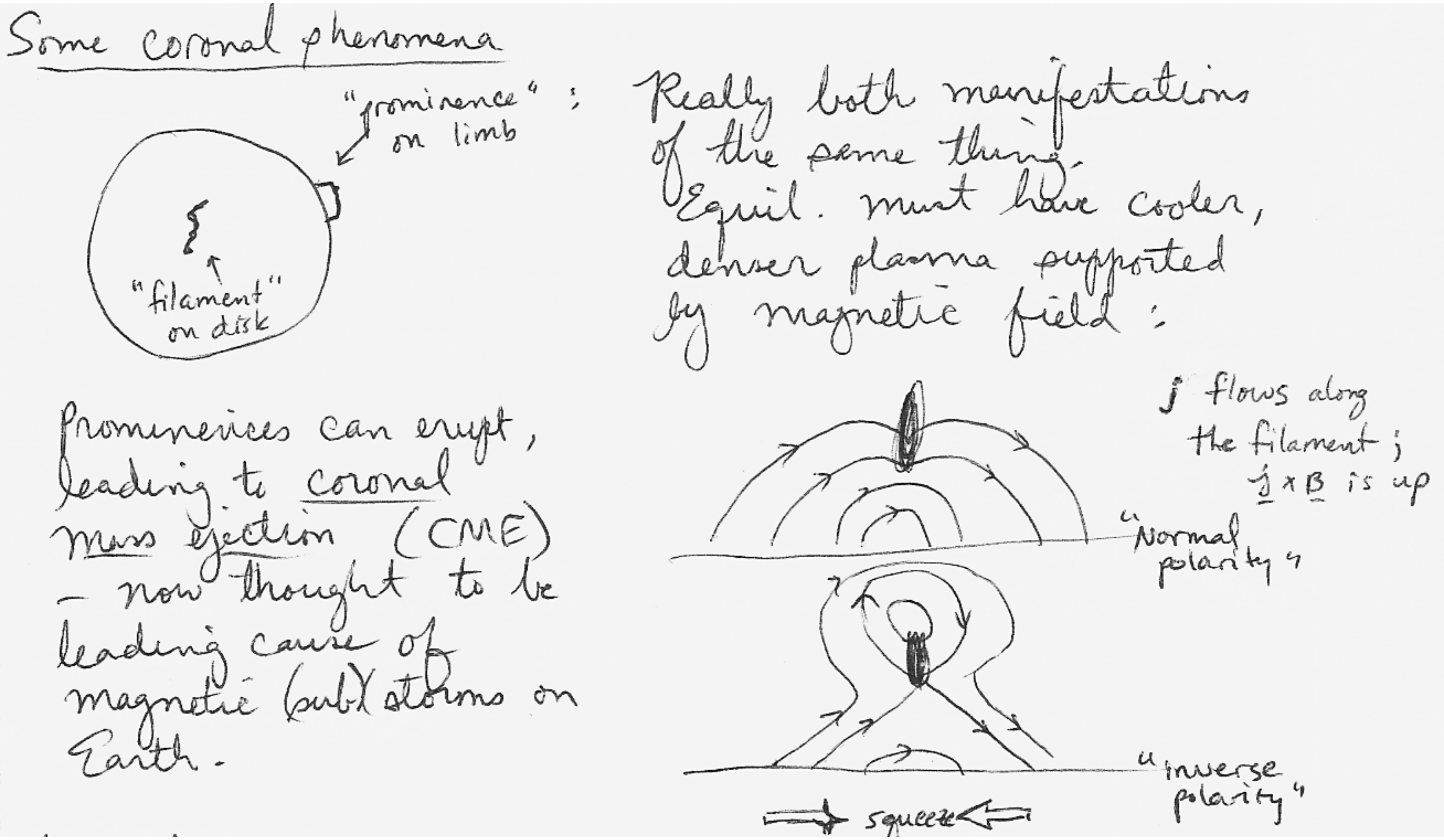
$\nabla \times \underset{\sim}{B} = \alpha \underset{\sim}{B}$ can have α = const. or $\alpha = \alpha(\underset{\sim}{x})$

note $\alpha = 0$ ⇒ potential field.

Both types of configurations are important for modeling the corona. Typical problem: given photospheric $\underset{\sim}{B}$, construct 3D magnetic fields in corona.

Coronal Structures – 2

Prominences and their eruption



Some coronal phenomena

Really both manifestations of the same thing.

Equil. must have cooler, denser plasma supported by magnetic field:

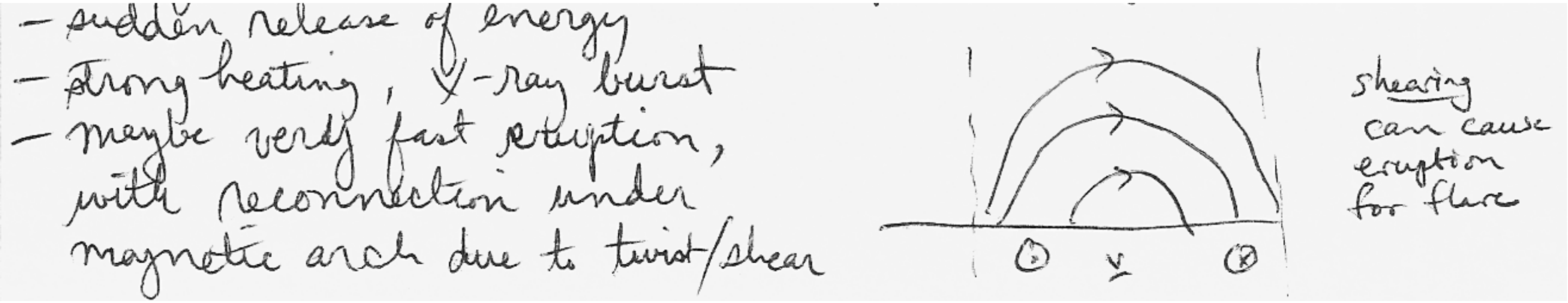
Prominences can erupt, leading to coronal mass ejection (CME) – now thought to be leading cause of magnetic (sub)storms on Earth.

j flows along the filament; $\underline{j} \times \underline{B}$ is up

Can get the prominence to eject by *squeezing* the footpoints

Coronal Structures – 3

Creating a solar flare



- sudden release of energy
- strong heating, X-ray burst
- maybe very fast eruption, with reconnection under magnetic arch due to twist/shear

shearing can cause eruption for flare

Can get the arcade to flare by *shearing* or *twisting* the footpoints

Coronal structures and dynamics can have consequences for Earth…

- Equilibrium structures (prominences, arcades) can suddenly lose stability, ejecting plasma and/or radiation into interplanetary space
- Low-level disturbances (waves, nanoflares) apparently heat the steady-state corona to high temperatures
  - This turns the corona into a much stronger X-ray source than the photosphere
  - As we will see, it drives a steady-state plasma outflow, the *solar wind*

Solar Wind Formation

First look at hydrostatic, extended corona. $\underline{v} = 0$ and pressure force is balanced by gravity:

$$-\frac{dp}{dr} - \rho\frac{GM_\odot}{r^2} = 0$$

assume $p = \rho\frac{kT}{\bar{m}}$

$T =$ const., $\bar{m} = 0.5\, m_p$ (half H^+, half e^-)

$$\frac{1}{p}\frac{dp}{dr} = -\frac{GM_\odot m_p}{2kT}\cdot\frac{1}{r^2}$$

$$\Rightarrow \ln p = \frac{GM_\odot m_p}{2kT}\frac{1}{r} + K$$

let $p = p_0$ at $r = R$, the base of the corona

$$\ln\frac{p}{p_0} = \frac{GM_\odot m_p}{2kT}\left(\frac{1}{r} - \frac{1}{R}\right)$$

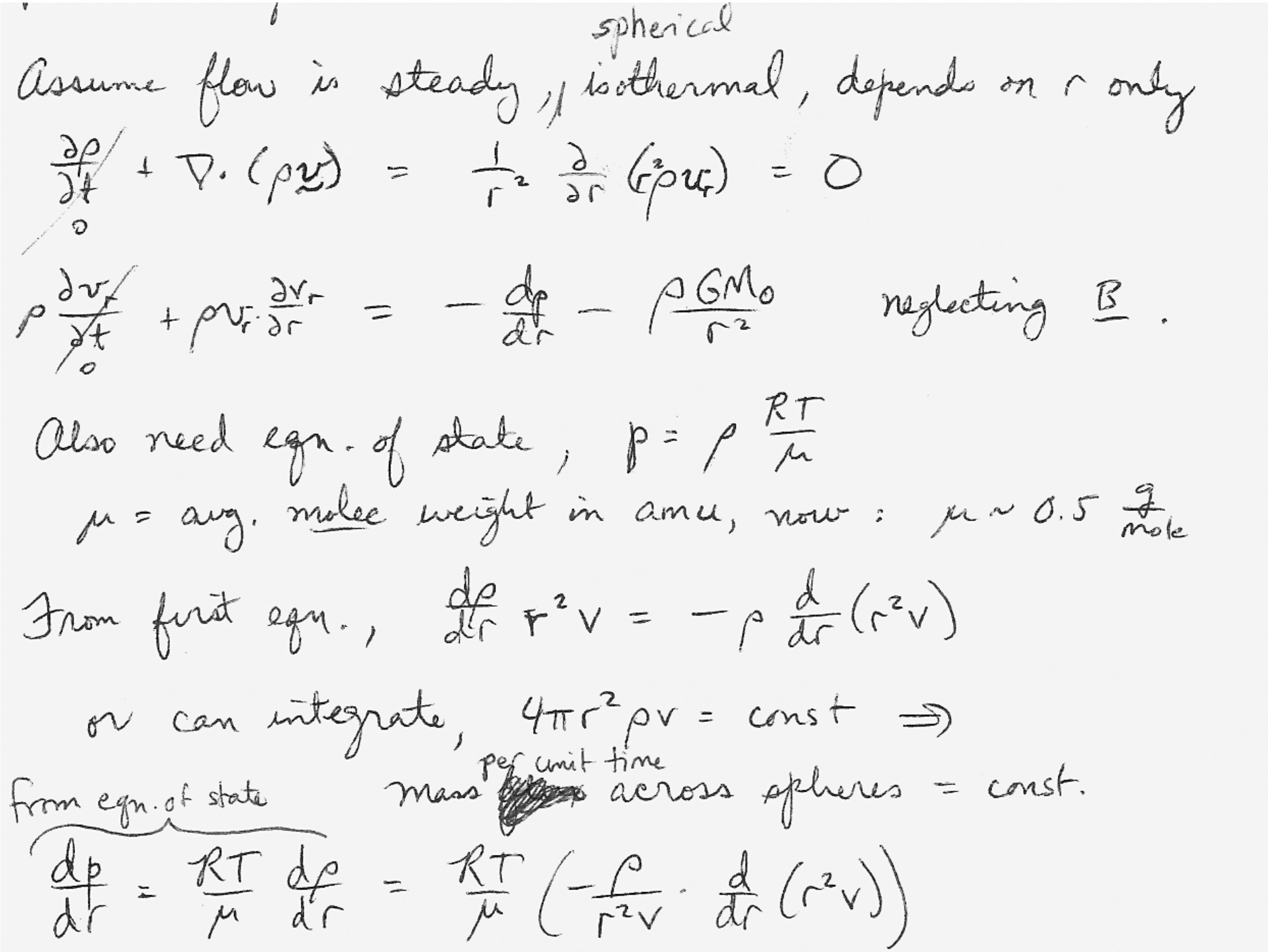
$$\boxed{p(r) = p_0 \exp\left\{\frac{GM_\odot m_p}{2kT}\left(\frac{1}{r} - \frac{1}{R}\right)\right\}}$$

hydrostatic sol'n.

problem: as $r \to \infty$, $p \to p_0 \exp\left\{-\frac{GM_\odot m_p}{2kTR}\right\}$

for coronal T of 10^6, this is $\sim p_0 e^{-8} \sim 3\times10^{-4}\, p_0$, far higher than p of ISM: mismatch.

Parker (1958) Solar Wind Equation – 1



Assume flow is steady, spherical isothermal, depends on r only

$$\cancel{\frac{\partial \rho}{\partial t}}_{0} + \nabla\cdot(\rho \underset{\sim}{v}) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho v_r) = 0$$

$$\rho \cancel{\frac{\partial v_r}{\partial t}}_{0} + \rho v_r \frac{\partial v_r}{\partial r} = -\frac{dp}{dr} - \rho\frac{GM_\odot}{r^2} \qquad \text{neglecting } \underline{B}.$$

Also need eqn. of state, $p = \rho\frac{RT}{\mu}$

μ = avg. <u>molec</u> weight in amu, now: $\mu \sim 0.5 \frac{g}{mole}$

From first eqn., $\frac{d\rho}{dr} r^2 v = -\rho \frac{d}{dr}(r^2 v)$

or can integrate, $4\pi r^2 \rho v = \text{const} \Rightarrow$

mass per unit time across spheres = const.

$$\overbrace{\frac{dp}{dr} = \frac{RT}{\mu}\frac{d\rho}{dr}}^{\text{from eqn. of state}} = \frac{RT}{\mu}\left(-\frac{\rho}{r^2 v}\cdot\frac{d}{dr}(r^2 v)\right)$$

Parker (1958) Solar Wind Equation – 2

$$v \frac{dv}{dr} = + \frac{RT}{\mu} \frac{1}{r^2 v} \frac{d}{dr}(r^2 v) - \frac{GM_\odot}{r^2}$$

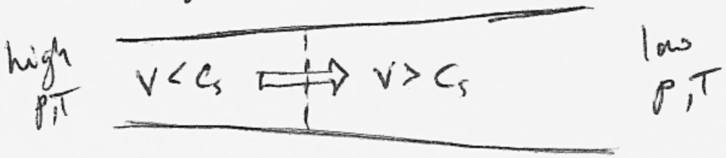
$$= \frac{RT}{\mu} \frac{2\cancel{r}\cancel{v}}{r^{\cancel{2}}\cancel{v}} + \frac{RT}{\mu} \frac{1}{v} \frac{dv}{dr} - \frac{GM_\odot}{r^2}$$

$$\left(1 - \frac{RT}{\mu v^2}\right) v \frac{dv}{dr} = \frac{2RT}{\mu} \frac{1}{r} - \frac{GM_\odot}{r^2}$$

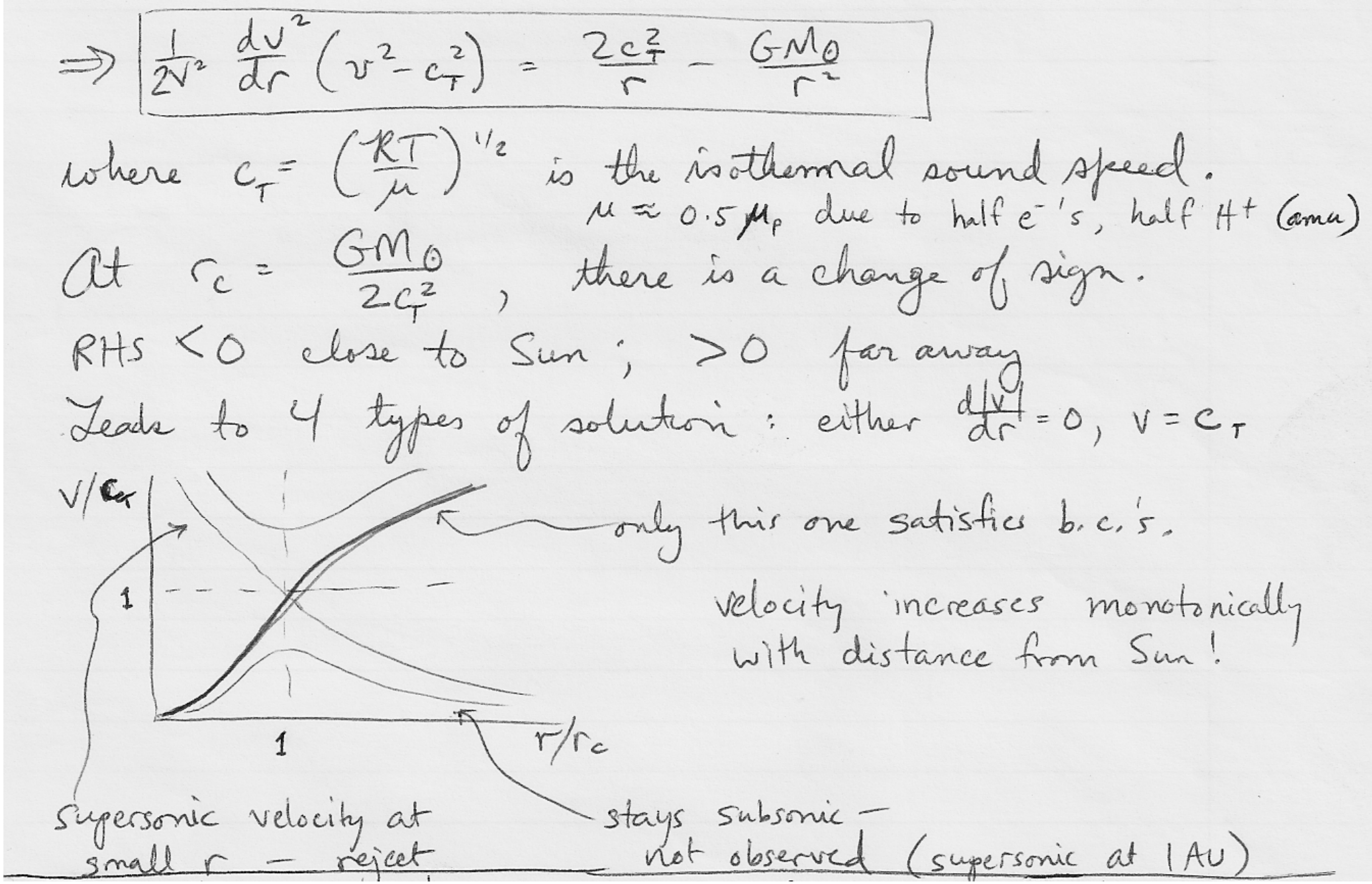
Define $c_T^2 = RT/\mu$, like a sound speed. $c_s^2 = \frac{\gamma RT}{\mu}$

Goal: solutions for which $\rho = \frac{\text{const.}}{4\pi r^2 v} \to 0$, $p \to 0$ for large r. This says $v \sim r^q$ where $q > -2$

But also: need transonic flow solution (sub → super) like flow in a Laval nozzle – diverging geometry



Parker (1958) Solar Wind Equation – 3



$$\Rightarrow \boxed{\frac{1}{2v^2}\frac{dv^2}{dr}\left(v^2-c_T^2\right) = \frac{2c_T^2}{r} - \frac{GM_\odot}{r^2}}$$

where $c_T = \left(\frac{RT}{\mu}\right)^{1/2}$ is the isothermal sound speed.

$\mu \approx 0.5\,\mu_p$ due to half e^-'s, half H^+ (amu)

At $r_c = \frac{GM_\odot}{2c_T^2}$, there is a change of sign.

RHS < 0 close to Sun; > 0 far away

Leads to 4 types of solution: either $\frac{d|v|}{dr} = 0$, $v = c_T$

only this one satisfies b.c.'s.

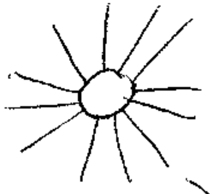
velocity increases monotonically with distance from Sun!

supersonic velocity at small r – reject

stays subsonic – not observed (supersonic at 1 AU)

The subsonic “solar breeze” solution is also permitted but is not observed

Spiral Magnetic Field in the Solar Wind – 1



Consider a non-rotating sun + solar wind. Magnetic field lines would be blown radially outwards until straight – only steady state possible (field lines join at ∞).

Rotating sun: still have $\underline{u} \parallel \underline{B}$ in rotating frame of reference (field lines are anchored to rotating sun). If $\underline{u}$ is isn't $\parallel \underline{B} \Rightarrow \underline{B}$ varies in time.

Assuming u_φ is zero in fixed frame, then in rotating frame, $u'_\varphi = -\omega r$. (Note that this is a weird assumption. Normally, if you pitched something off a merry-go-round, e.g., it would have $u_\varphi = +\omega r$, $u'_\varphi = 0$. Here things are constructed so that the angular momentum of the solar wind is zero.) (You push it off backward, so you speed up the sun by some tiny amount!)

Spiral Magnetic Field in the Solar Wind – 2

Let's try several assumptions.

(1) $u_\varphi' = -\omega r$. Easiest to compute. at $r=R$

(2) $u_\varphi'(R)=0$, i.e., velocity component in fixed
frame is $+\omega R$, instantaneous vel. at radius
R where solar wind parcel leaves Sun [then conserves angular mmtm.

(1) Show that field lines are Archimedean spirals—the same pattern made by streams of water from a rotating lawn sprinkler when viewed from above:

OK: $(1)^*$ $\underline{u} \parallel \underline{B} \Rightarrow \underline{u} \times \underline{B} = 0 \Rightarrow u_r B_\varphi - u_\varphi B_r = 0$

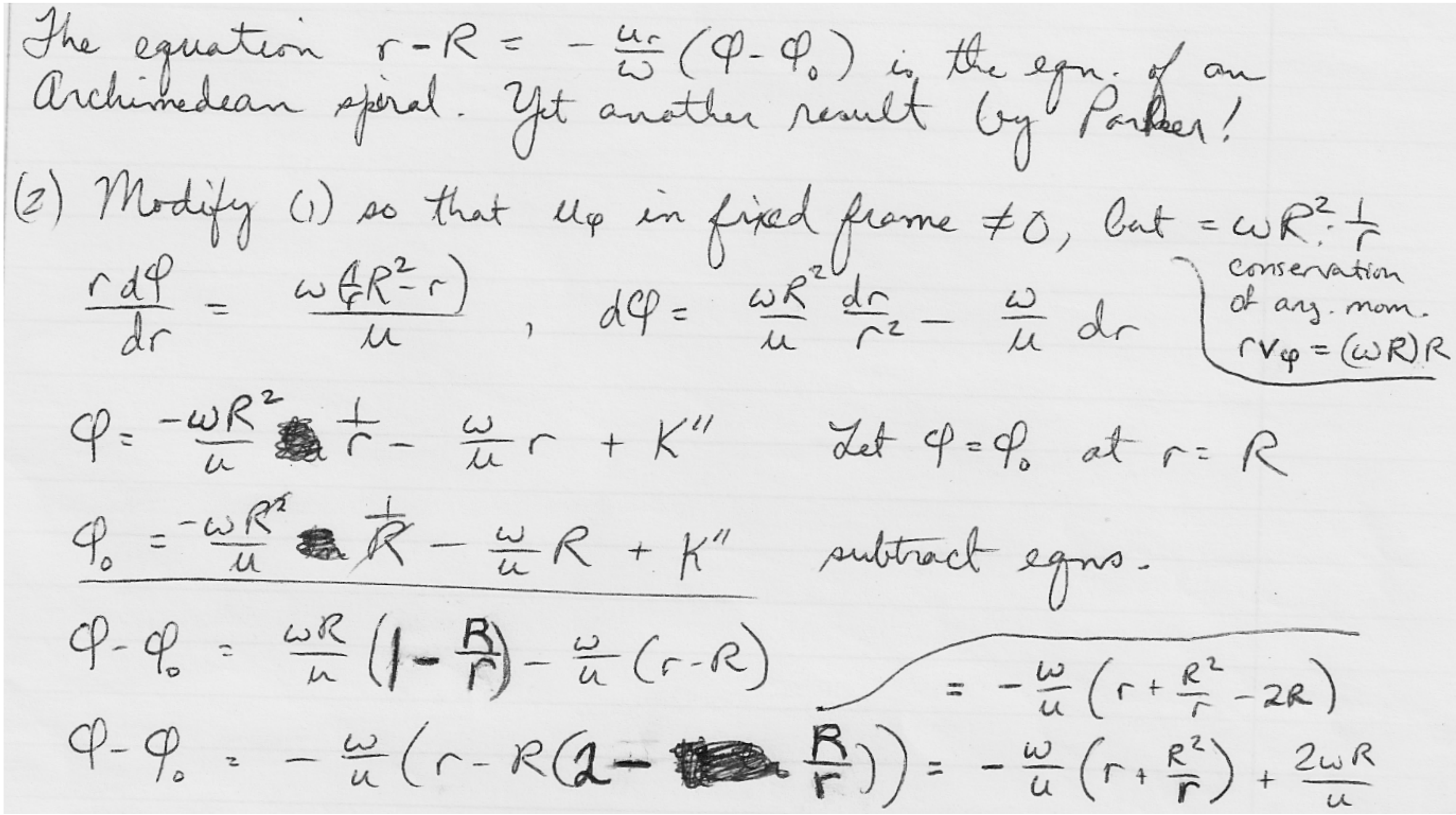
$\therefore \dfrac{u_\varphi}{u_r} = \dfrac{B_\varphi}{B_r}$ *(have dropped the primes). Let $u_r =$ const.

$-\dfrac{\omega r}{u_r} = \dfrac{B_\varphi}{B_r}$. Eqn. of field line: $\dfrac{r\,d\varphi}{B_\varphi} = \dfrac{dr}{B_r}$

$\therefore \dfrac{r\,d\varphi}{dr} = -\dfrac{\omega r}{u_r}$, $d\varphi = -\dfrac{\omega}{u_r}\,dr$, $\boxed{\varphi - \varphi_0 = -\dfrac{\omega}{u_r}(r-R)}$

where φ is longitude of field line leave base of corona at R

Spiral Magnetic Field in the Solar Wind – 3



The equation $r - R = -\frac{u_r}{\omega}(\varphi - \varphi_0)$ is the eqn. of an Archimedean spiral. Yet another result by Parker!

(2) Modify (1) so that u_φ in fixed frame $\neq 0$, but $= \omega R^2 \cdot \frac{1}{r}$

$$\frac{r\,d\varphi}{dr} = \frac{\omega\left(\frac{1}{r}R^2 - r\right)}{u}, \quad d\varphi = \frac{\omega R^2}{u}\frac{dr}{r^2} - \frac{\omega}{u}\,dr$$

conservation of ang. mom. $rv_\varphi = (\omega R)R$

$$\varphi = -\frac{\omega R^2}{u}\frac{1}{r} - \frac{\omega}{u}r + K''$$

Let $\varphi = \varphi_0$ at $r = R$

$$\varphi_0 = -\frac{\omega R^2}{u}\frac{1}{R} - \frac{\omega}{u}R + K''$$

subtract eqns.

$$\varphi - \varphi_0 = \frac{\omega R}{u}\left(1 - \frac{R}{r}\right) - \frac{\omega}{u}(r - R)$$

$$= -\frac{\omega}{u}\left(r + \frac{R^2}{r} - 2R\right)$$

$$\varphi - \varphi_0 = -\frac{\omega}{u}\left(r - R\left(2 - \frac{R}{r}\right)\right) = -\frac{\omega}{u}\left(r + \frac{R^2}{r}\right) + \frac{2\omega R}{u}$$

Sum of an Archimedean spiral and a hyperbolic spiral

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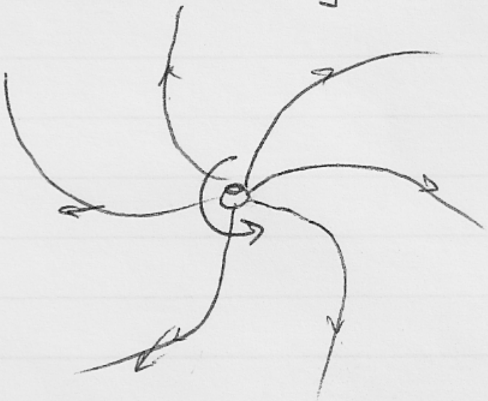
www.cac.cornell.edu/~slantz

Spiral Magnetic Field in the Solar Wind – 4

magnetic field is advected radially outwards — but Sun is rotating — combines to give

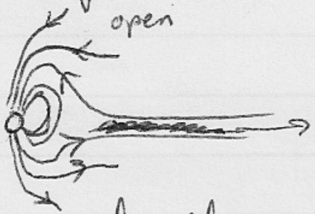


Spiral field lines.

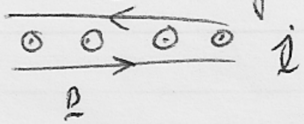
↕

"corotating streams"

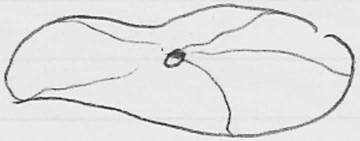
dipole field: lines oppositely directed in each hemisphere ⇒ current sheet *



cross sect. close-up

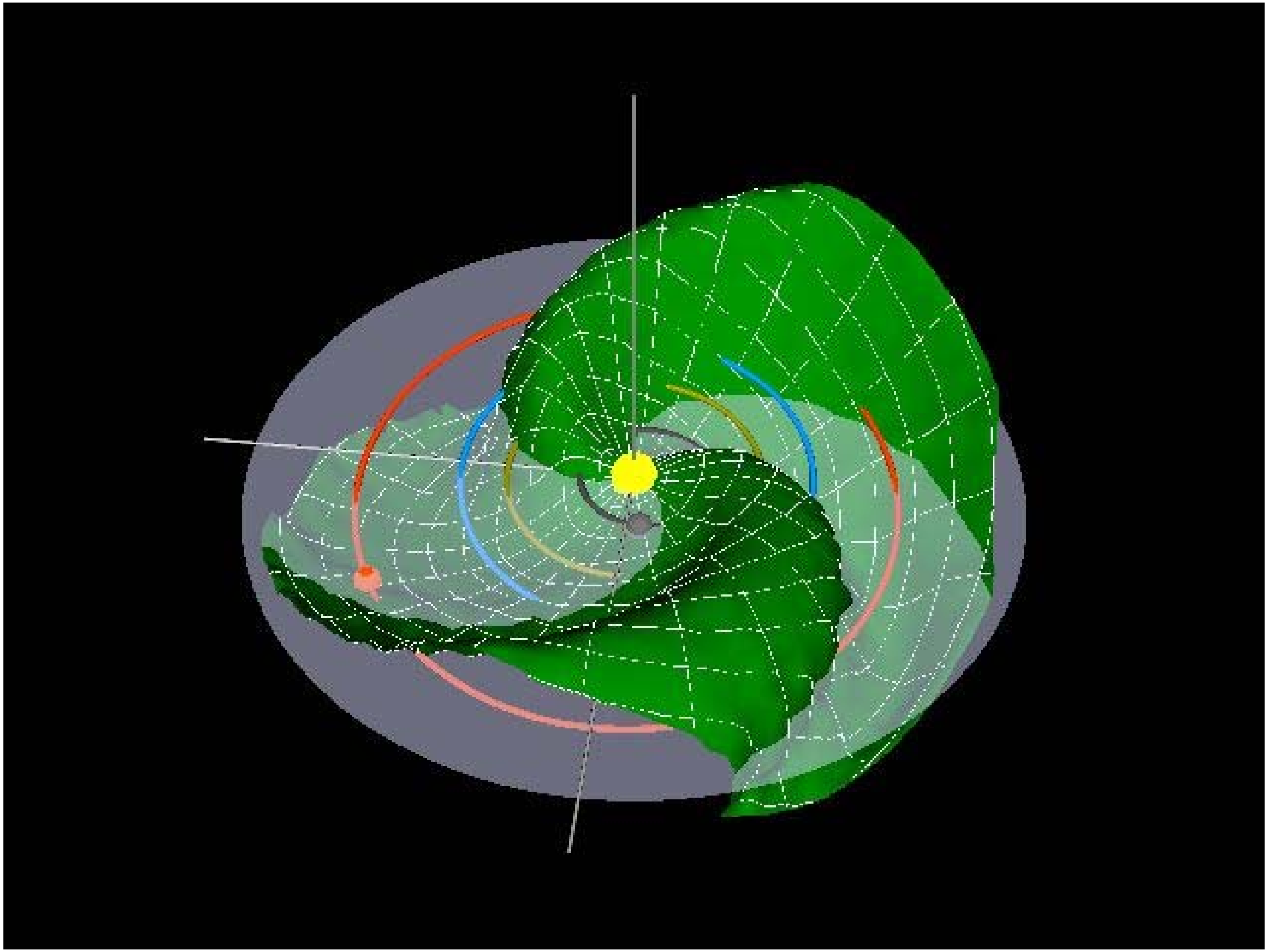


3-D:



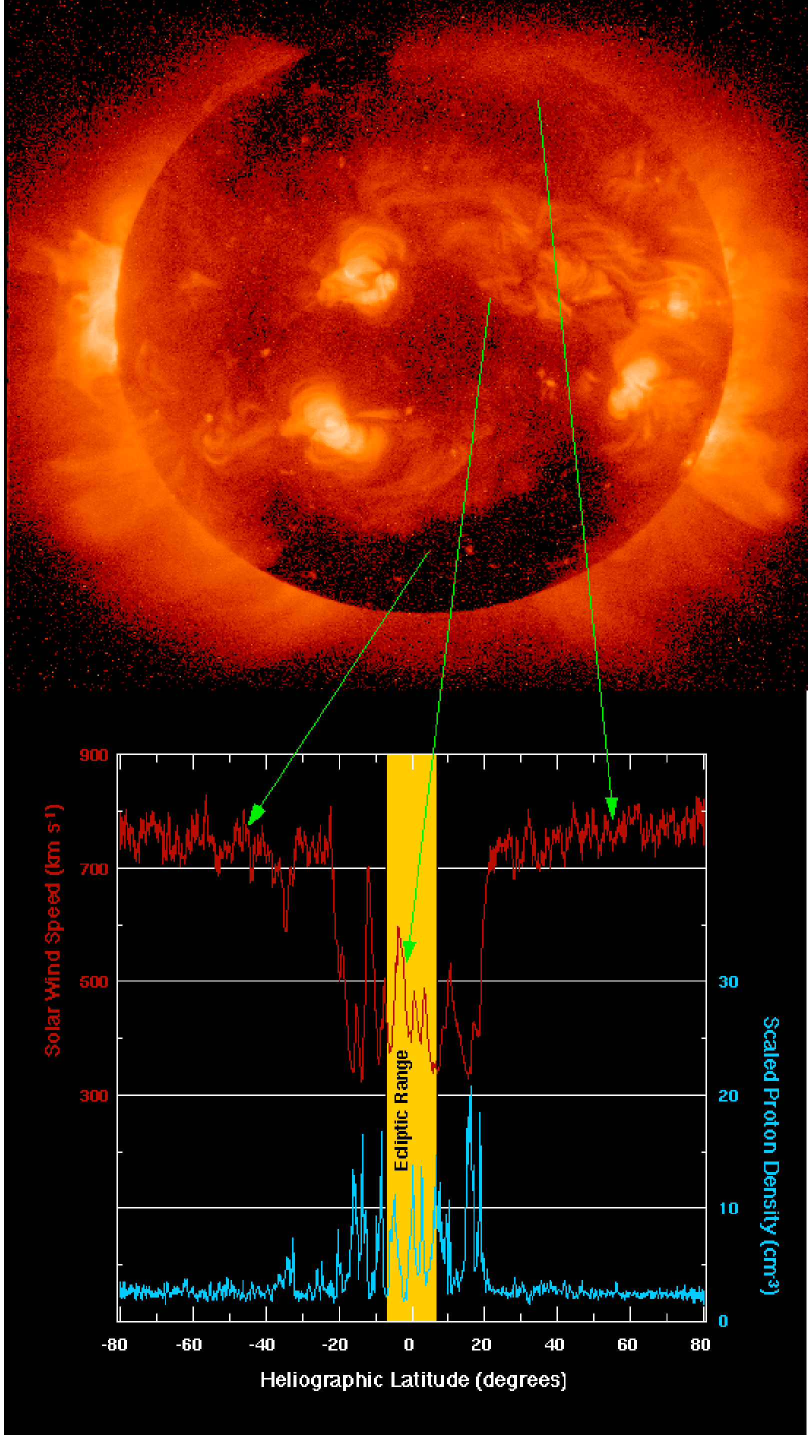
"hat with floppy brim"

The 3D Current Sheet: “Ballerina Skirt”



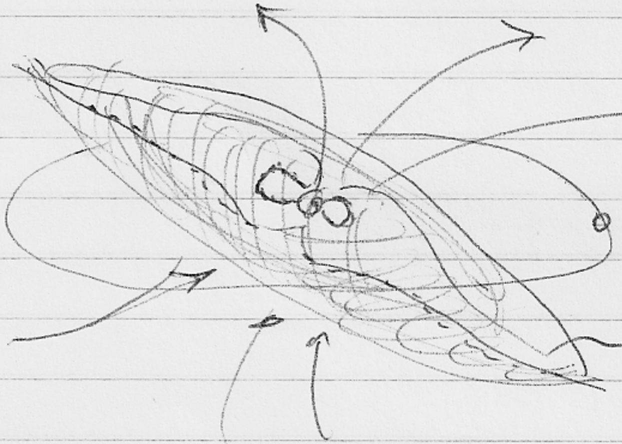
Ulysses Main Results

- There are two distinct plasma regimes in the solar wind
  - Near the equator, speed (red line) is low and density (blue line) is high. Composition is typical of the corona.
  - At high to mid latitudes, speed is high and density is low, with less variability in both. Composition is typical of the photosphere.
  - Speed is approximately 750 km/s everywhere except near the equator.
- The solar wind's magnetic field is not based on a dipole
  - A dipole field would be twice as strong over the poles; in the solar wind, it is near-uniform with latitude.



Spiral Magnetic Field in the Solar Wind – 5

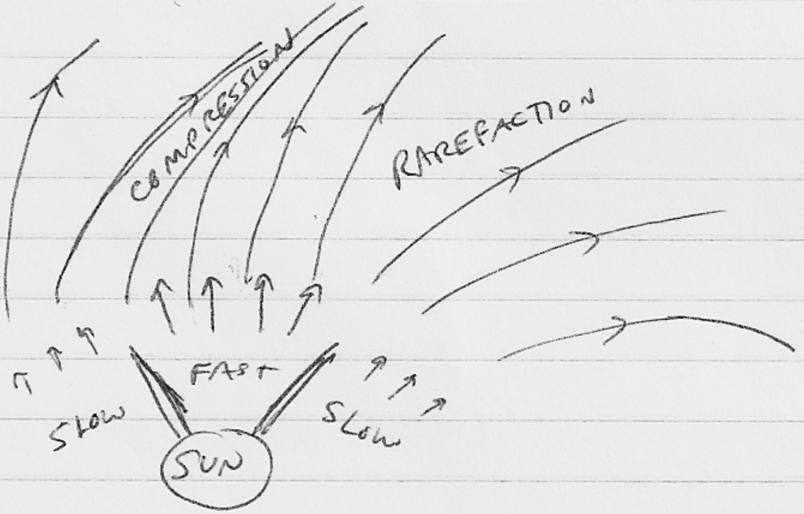
at 1 AU:



Open/fast region (extended holes)

2 "sector crossings" per solar rotation (inward/outward B)

slow/closed region (extended streamers)



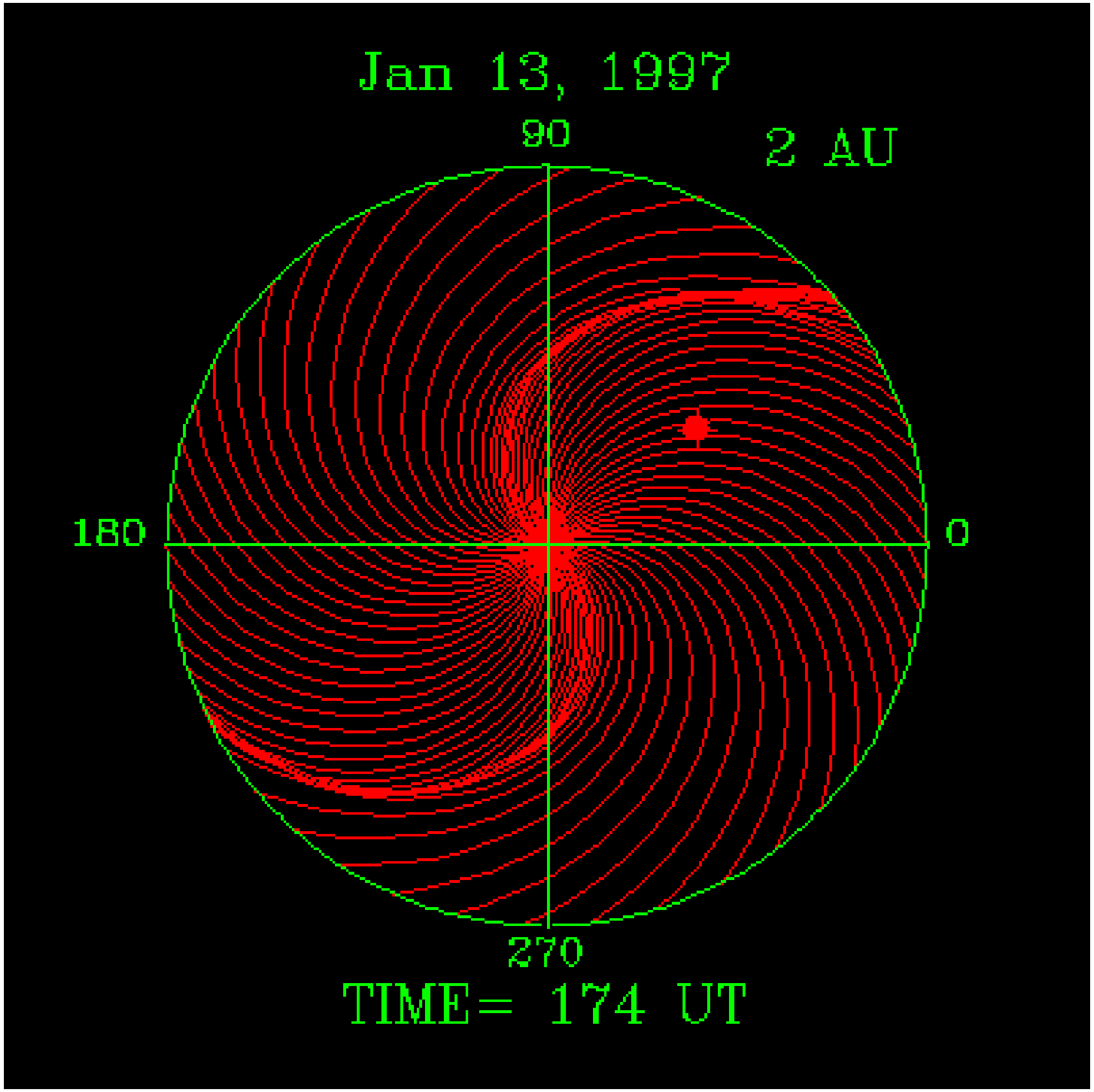
Fast streams create compressions in B, ρ, leading to shock fronts in distant solar wind.

quick rise

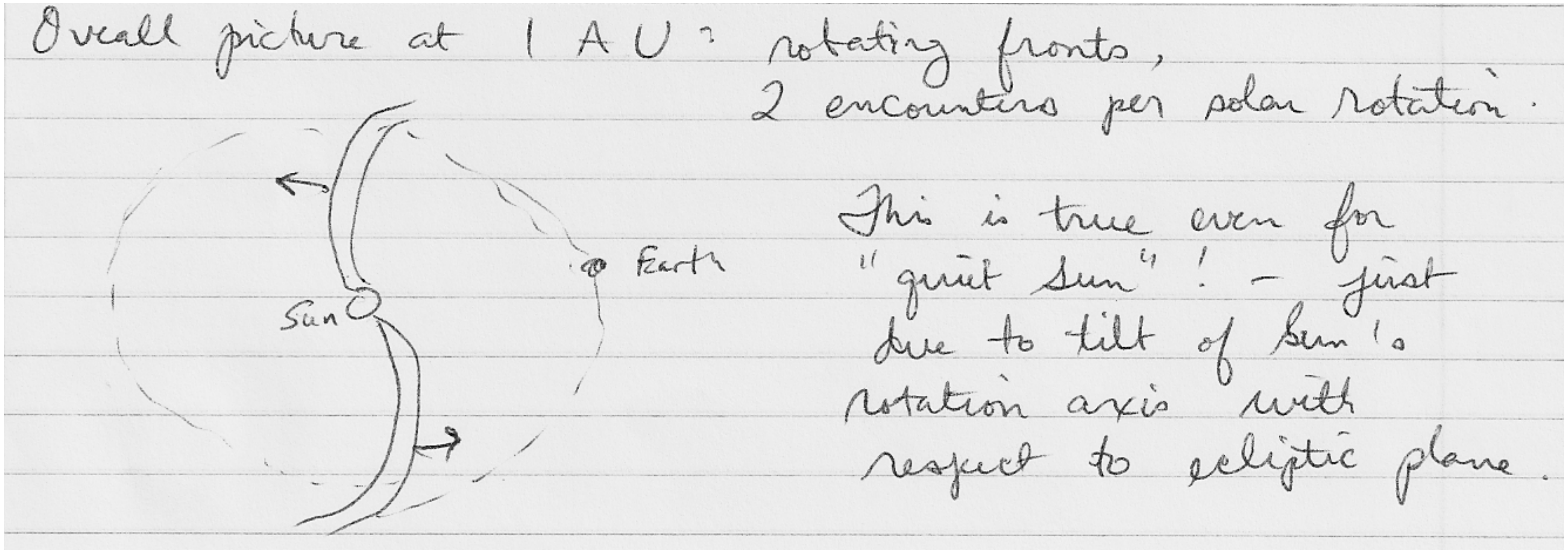
gradual fall-off (to rarefaction)

Shocks in the Interplanetary Medium

Where a fast corotating stream follows a slow one



Spiral Magnetic Field in the Solar Wind – 6



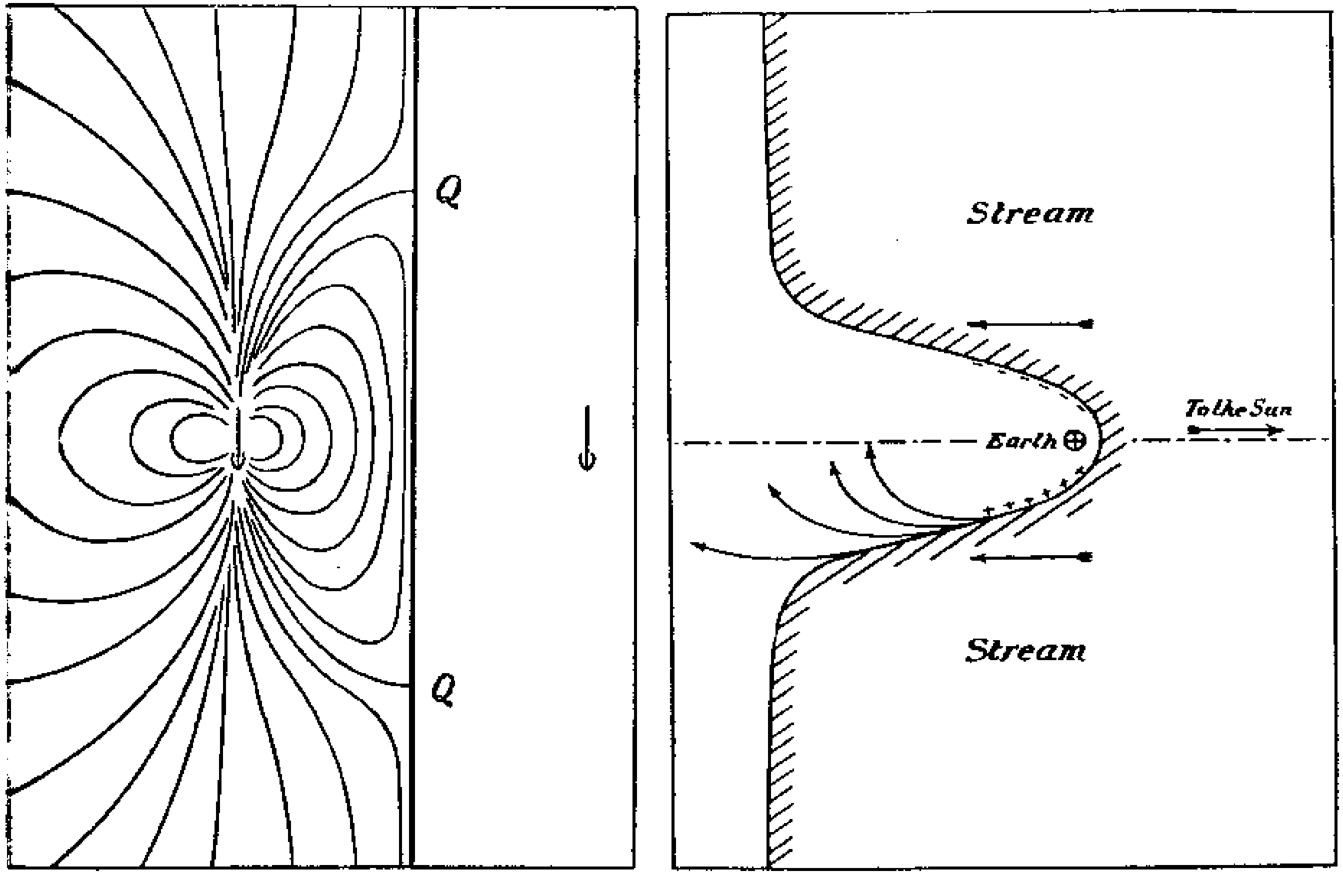
Overall picture at 1 AU: rotating fronts,
2 encounters per solar rotation.

This is true even for "quiet Sun"! – just due to tilt of Sun's rotation axis with respect to ecliptic plane.

- Sector crossings involving shock compression have a greater effect on geospace than those involving rarefactions
- To see why, first need to understand the steady state of interactions between the near-Earth environment and the solar wind…

Chapman-Ferraro (1930) Magnetosphere

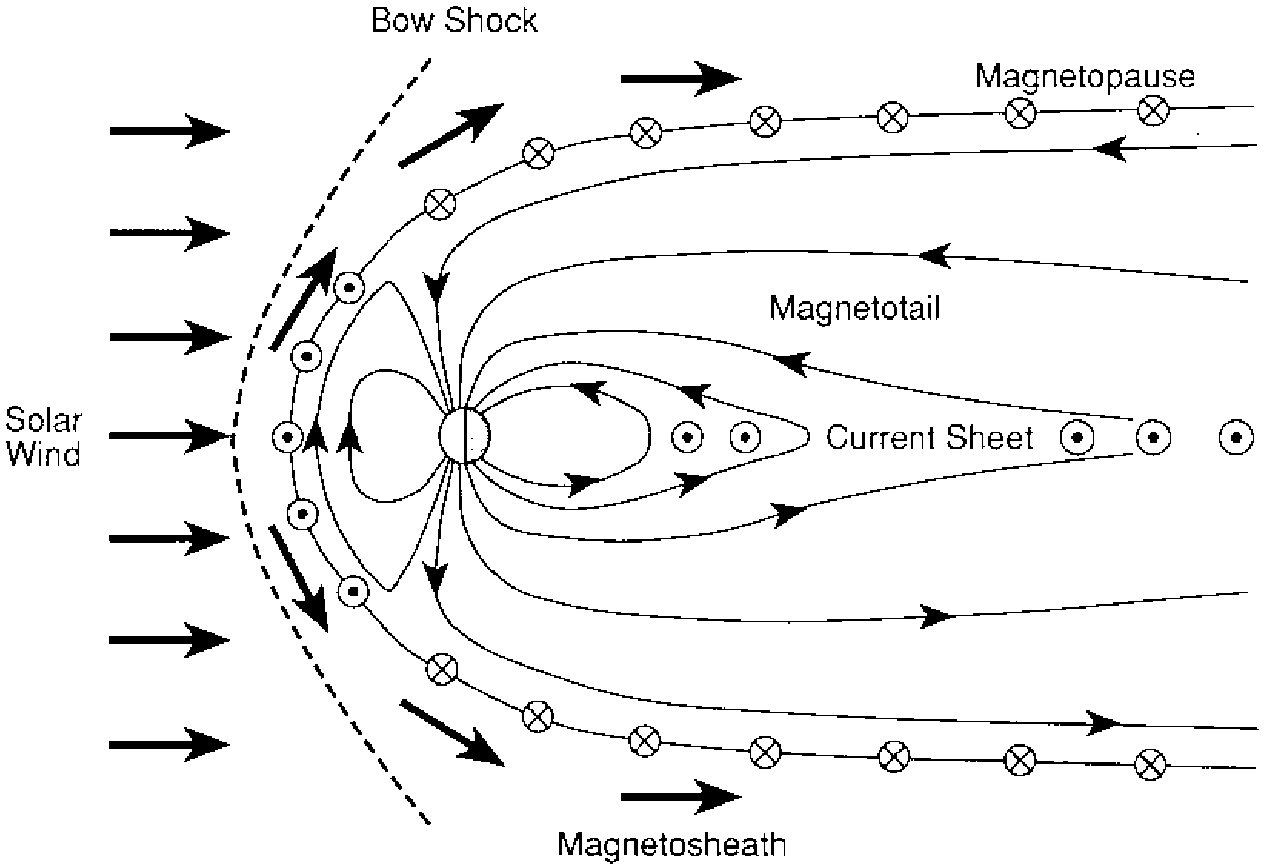
(Figure from Chapman and Bartels, 1940)



- Solar wind is like a superconductor that excludes the sunward dipole field
- At planar boundary, a current sheet forms; field is summed with image dipole
- Separatrix QQ defines the "cusp" latitude associated with auroral ovals

Anatomy of the Earth’s Magnetosphere

Current Sheets at the Magnetopause and Across the Tail

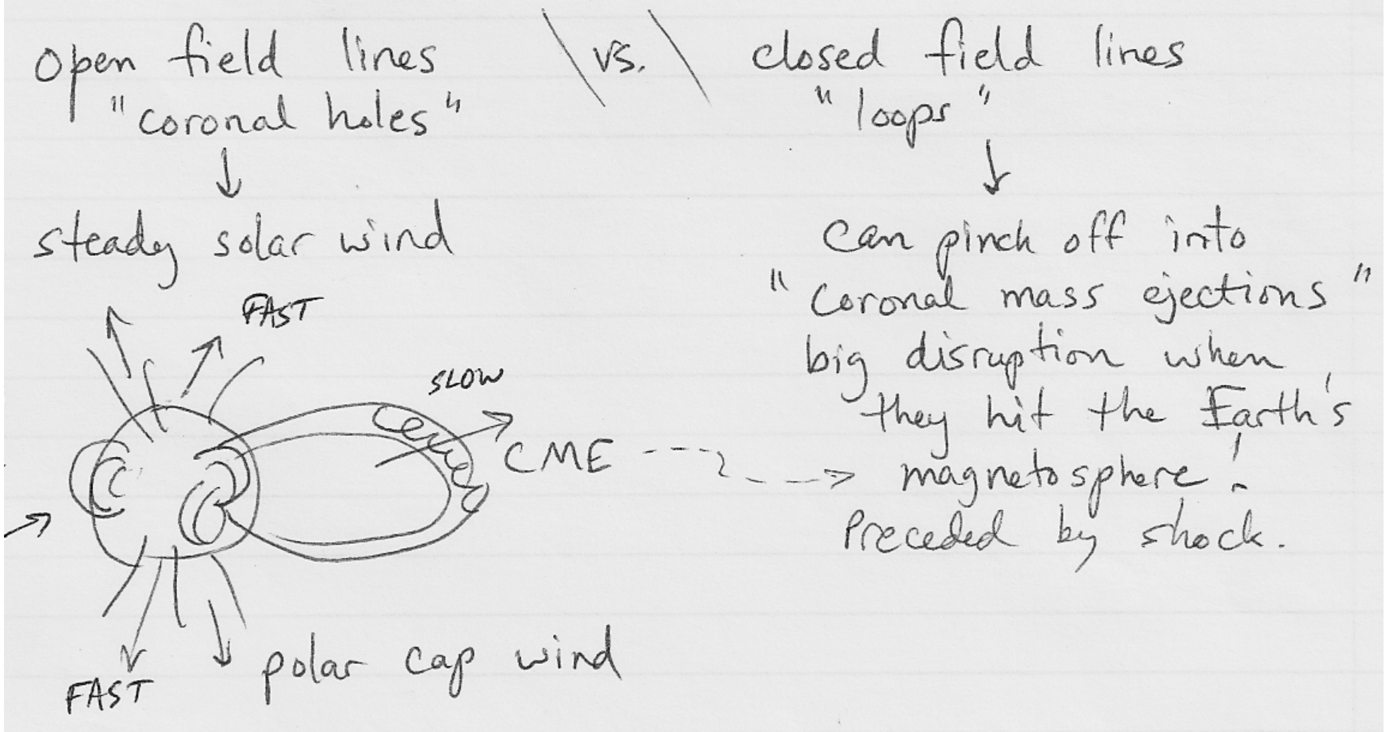


At the “nose” of the magnetosphere, the magnetic pressure of the Earth’s squeezed dipole field can stand up to the ram pressure of the solar wind

Figure from Kivelson and Russell

Coronal Mass Ejections

How to launch a “magnetic cloud”



open field lines vs. closed field lines

"coronal holes" "loops"

↓ ↓

steady solar wind

can pinch off into "coronal mass ejections"

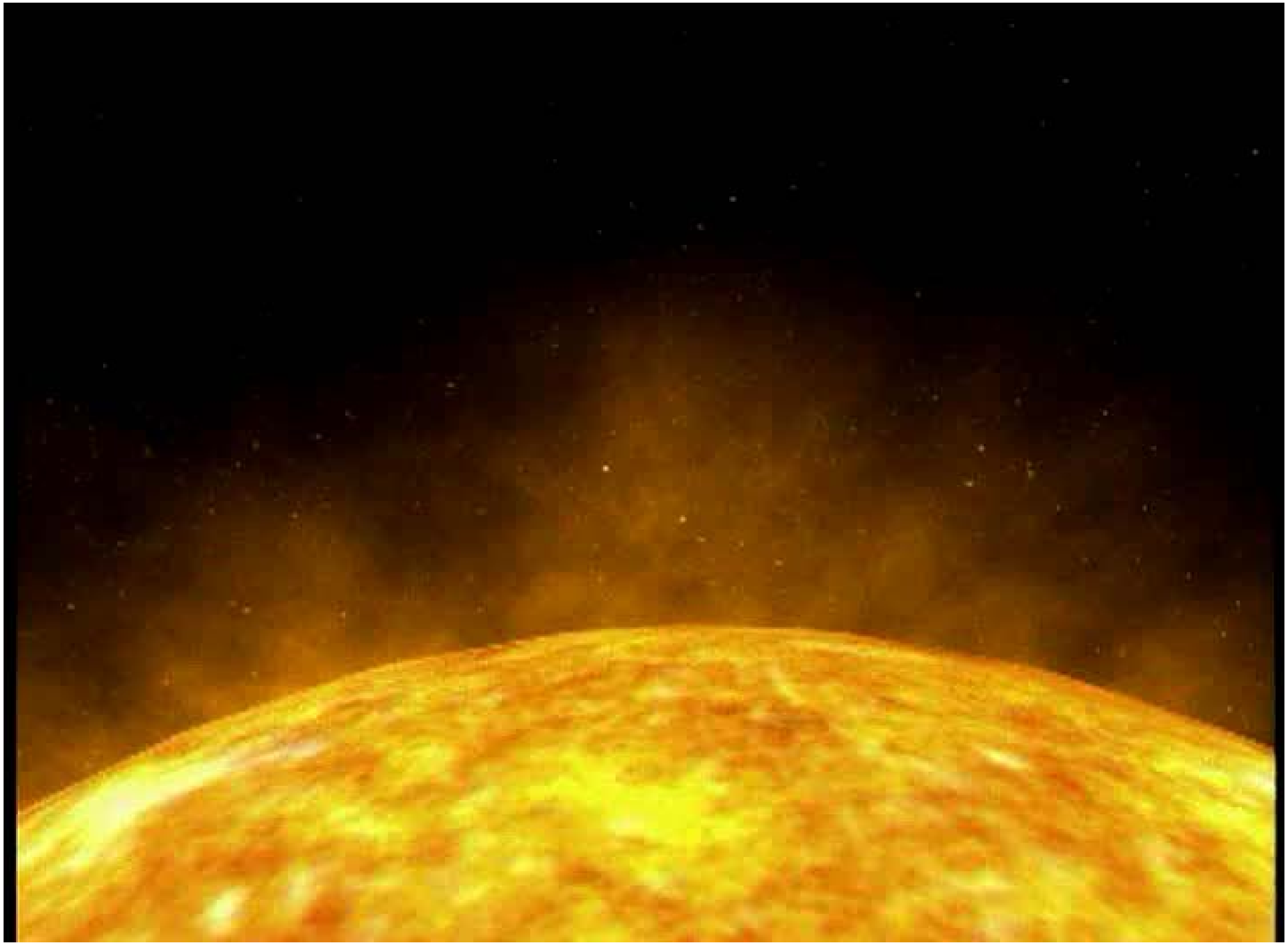
big disruption when they hit the Earth's magnetosphere!

Preceded by shock.

If aimed at Earth, a CME drastically changes the momentum (velocity and density) of the solar wind that impinges on the magnetosphere

Magnetic Reconnection and Plasmoid Ejection

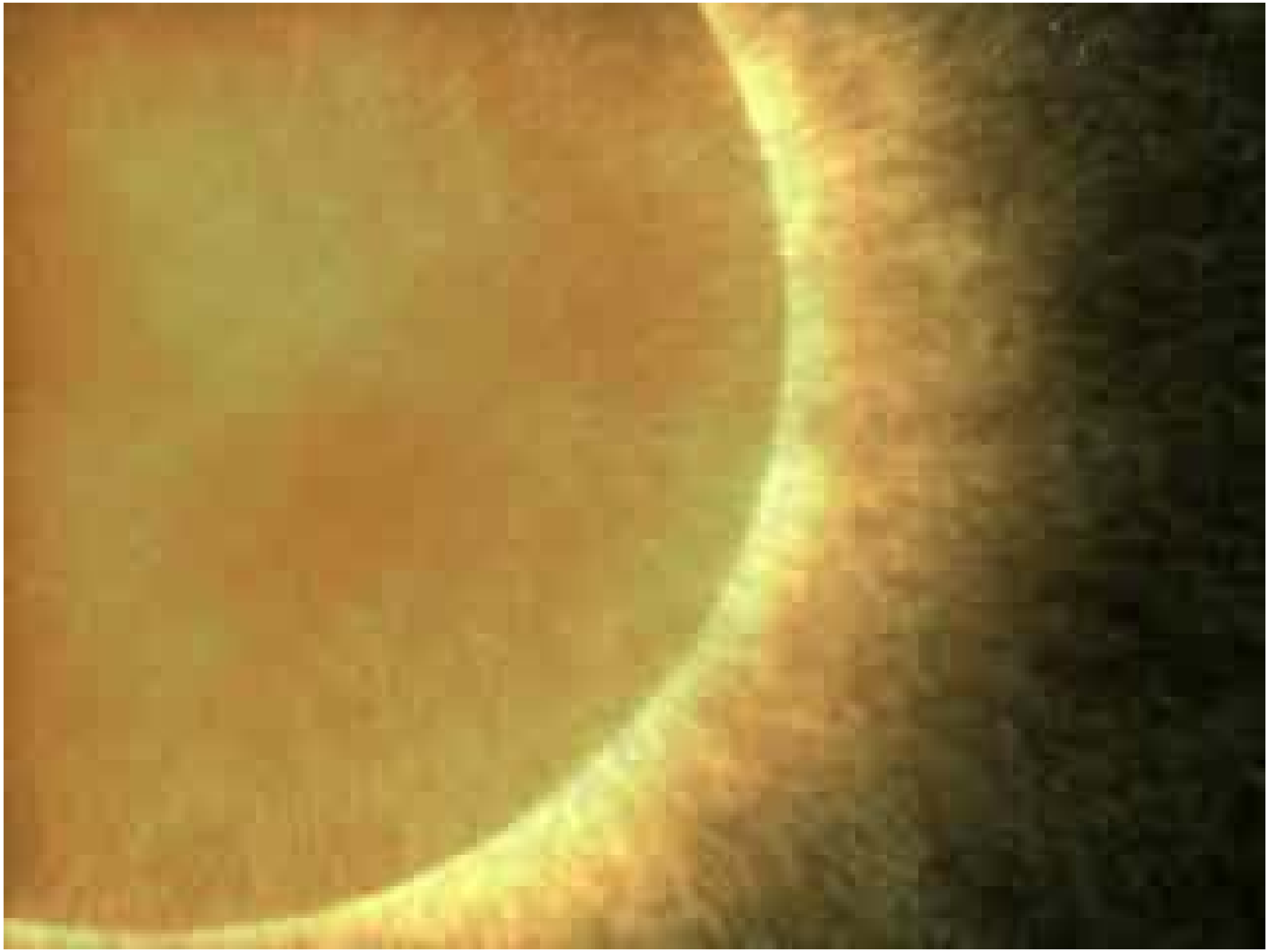
Magnetic could contains *southward* IMF



**Play QuickTime movie of solar wind gusts hitting the magnetosphere*

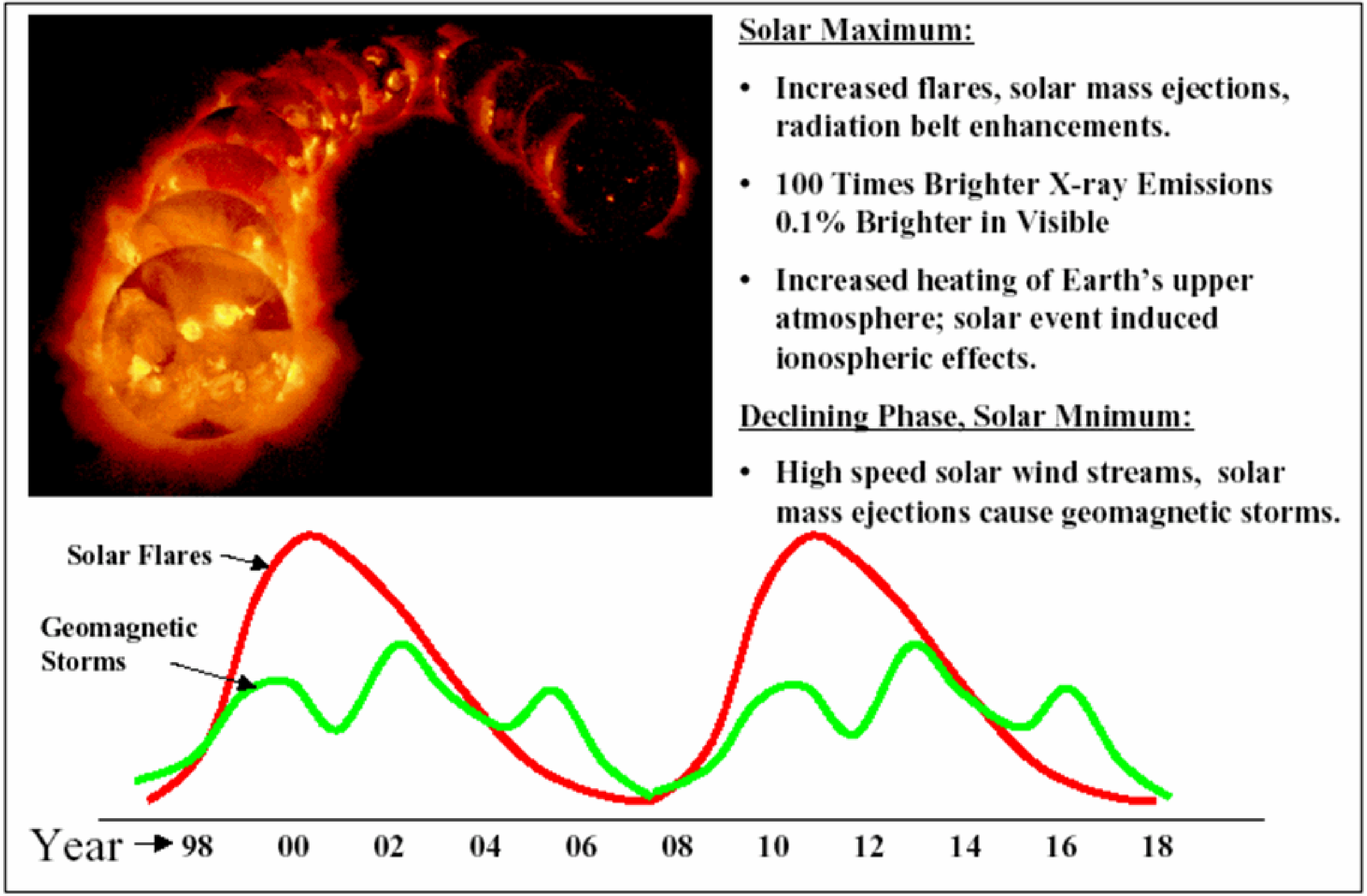
Cusp Aurora Due to Reconnection at High Latitude

Magnetic cloud contains *northward* IMF



http://web.ift.uib.no/~nikost/research.html

Sun-Earth System Is Driven by the 11-Year Solar Cycle



Solar Maximum:

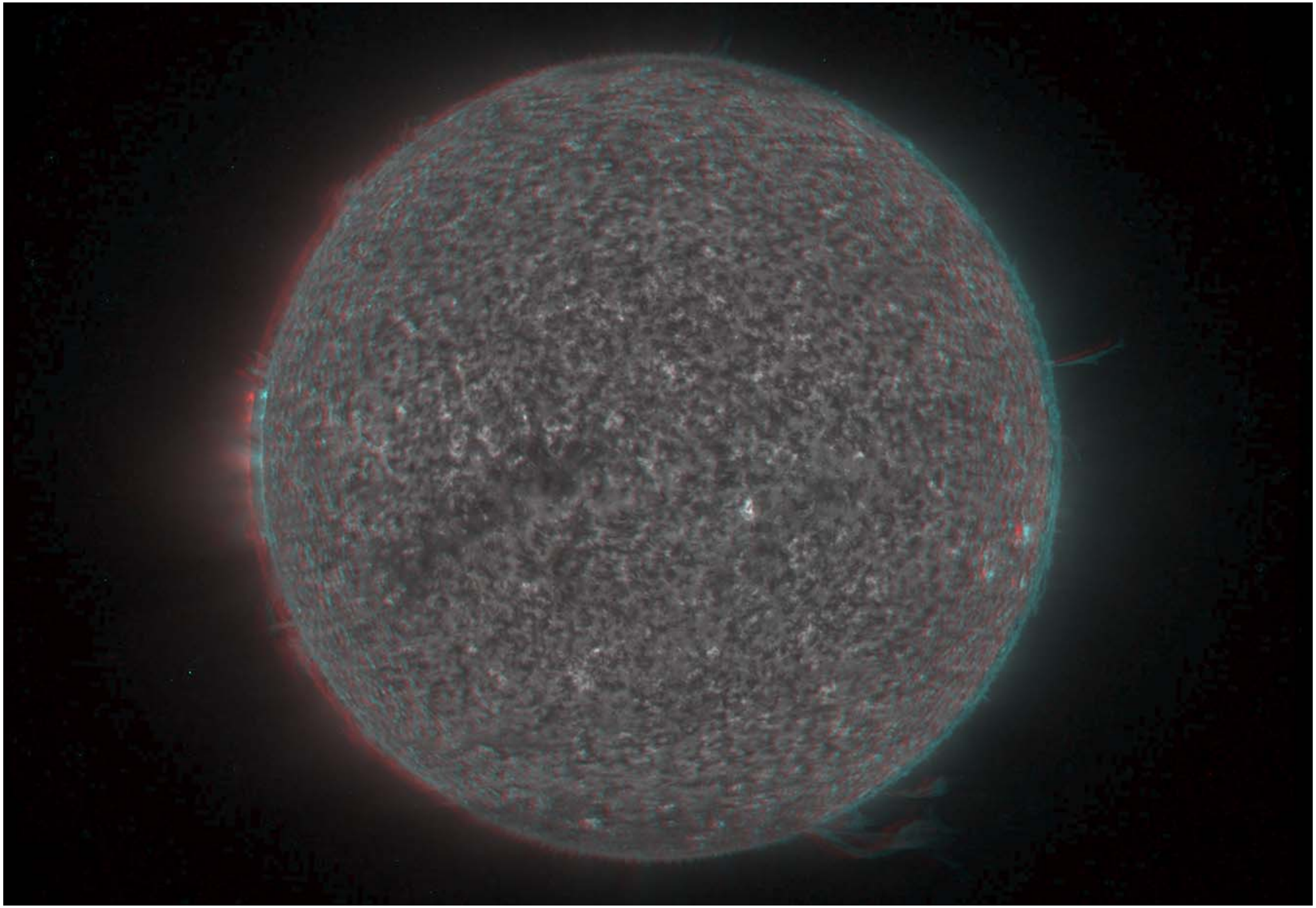
- Increased flares, solar mass ejections, radiation belt enhancements.
- 100 Times Brighter X-ray Emissions 0.1% Brighter in Visible
- Increased heating of Earth's upper atmosphere; solar event induced ionospheric effects.

Declining Phase, Solar Mnimum:

- High speed solar wind streams, solar mass ejections cause geomagnetic storms.

First-Ever 3D Images of the Sun from STEREO –

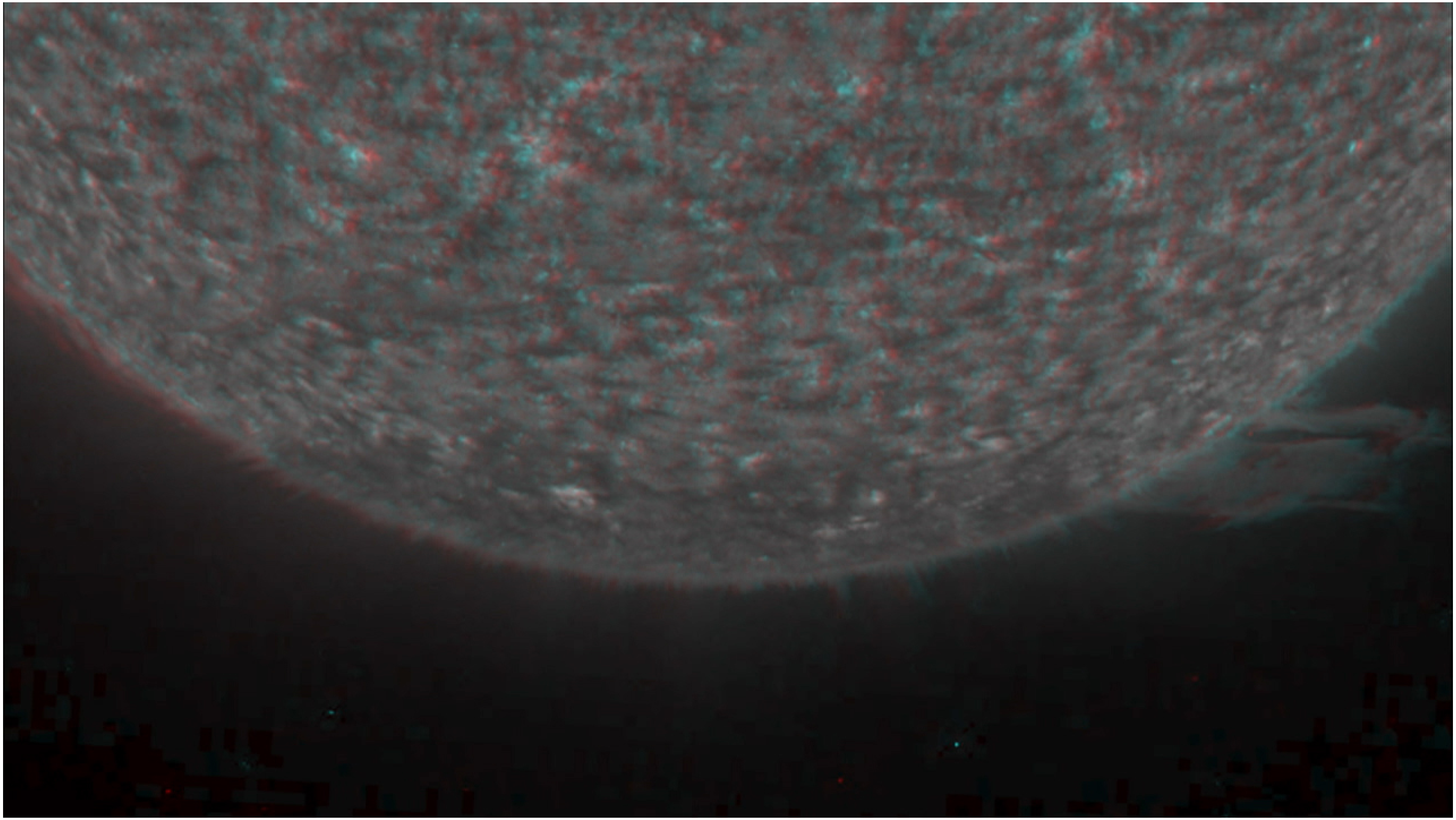
NASA's Solar TErrestrial RElations Observatory satellites



http://www.nasa.gov/mission_pages/stereo/news/stereo3D_press.html

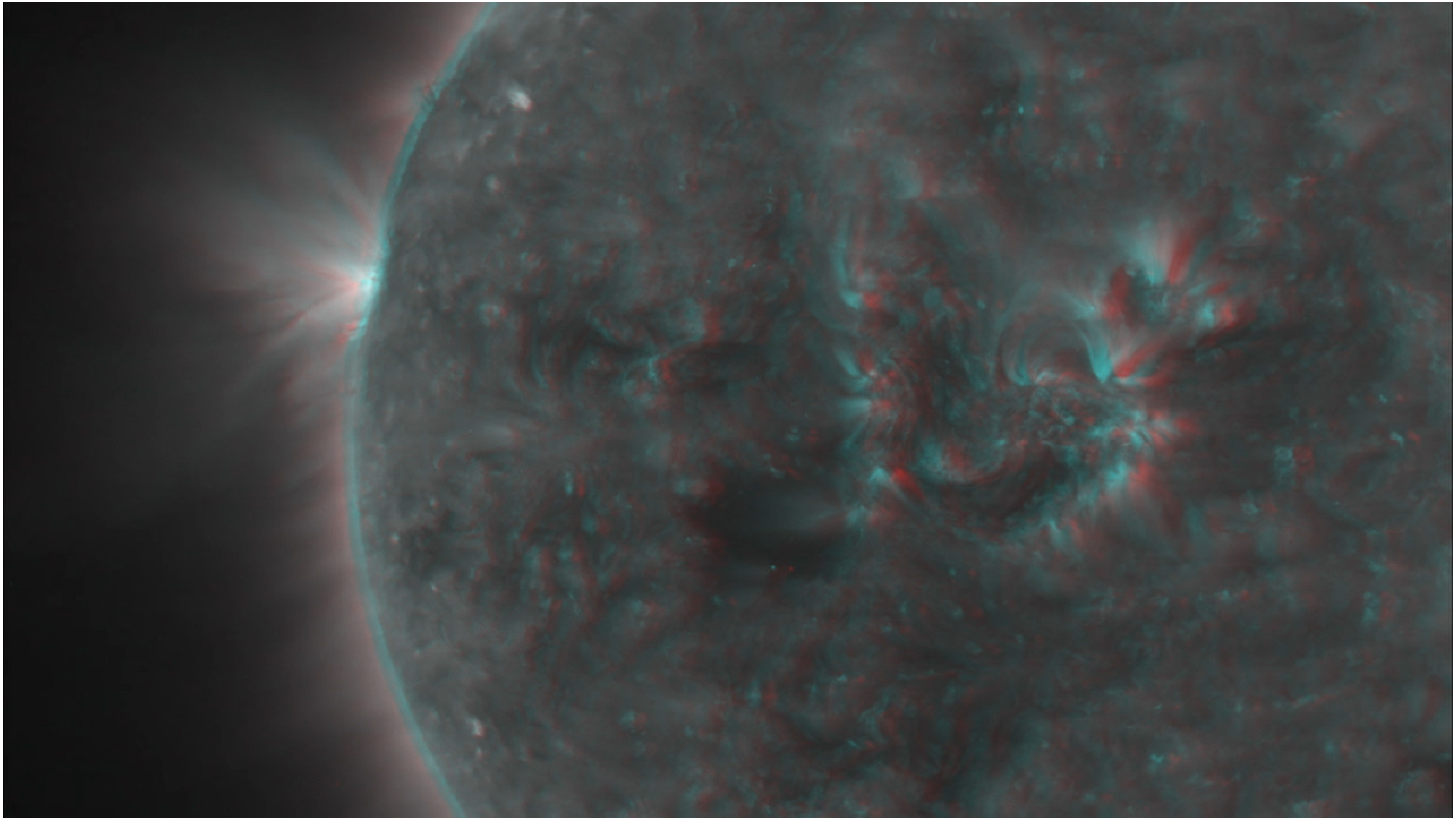
STEREO Images – 2

Spicules, Polar Coronal Hole, Prominence



STEREO Images – 3

Active Regions



**See QuickTime movies for 3D animations*